TIME DELAY ESTIMATION OF SIGNALS USING WIGNER-VILLE DISTRIBUTION

BY

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DECLARATION

I hereby declare that this thesis “time delay estimation of signals using quadratic time frequency” distribution has been written by me and it is a report of my research work. It has not been presented in any previous application for Doctor of Philosophy Degree. All quotations are indicated and sources of information specifically acknowledged by means of references.

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CERTIFICATION

This thesis *Time Delay Estimation of Signals Using Wigner Ville Distribution* meets the regulations governing the award of Doctor of Philosophy Degree of the School of Postgraduate Studies, Nasarawa State University, Keffi, and is approved for its contribution to knowledge.

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DEDICATION

This thesis is dedicated to Almighty ALLAH and to my beloved parents Dr. A. D. Zubairu and Mrs. Khadijat Yahaya, for their love and blessing which have been the light to my way and all my achievements in life.

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ABSTRACT

The conventional representation of signals in the time domain or frequency domain are inadequate for analysis of signals, the obvious solution was to seek an alternative way to represent the signal as a two-variable function whose domain was two-dimensional (t, f)space. An effective method based on correlation was proposed in this research work for the estimation time-delay of a signal which is corrupted by the non-stationary random noise. The time delay estimation was converted into parameter estimation. The research shows that the concept of reducing cross-terms with the help of using appropriate parameter values, which are unfortunate mathematical artifacts in determination of Wigner-Ville Distribution (WVD) of a multi-component chirp signal. The research was carried out using Linear Frequency Modulated (LFM) (chirp) signal based on quadratic time-frequency domain. From the derivation as well as simulation result of WVD of a multi-component chirp signal, it was clearly shown that cross-term appears in the middle of the signals. Quadratic TFD (QTFD) is able to provide high time and frequency resolutions but suffer heavily from the cross-terms which cause inaccurate signal interpretation. If the signal characteristics are known, a kernel can be designed that can suppress the cross-terms while preserving the auto-terms which is very important in generating accurate TFD. However, this is not practical in a non-cooperative environment where the exact signal characteristics are unknown. Thus, a new adaptive directional ambiguity function Wigner-Ville distribution (ADAF-WVD) was developed that adaptively estimate the kernel parameters in the ambiguity domain based on the signal characteristics. Accurate TFRs are produced for all signals with IF estimation performance verified using Monte Carlo simulation meeting the requirements of the Cramer-Rao Lower Bound (CRLB) at SNR > 8dB. A time delay estimation of chirp signal based on correlation method is presented which is converted to parameter estimation with less computational complexity and is suitable for chirp signal analysis. The statistical analysis in terms of signal-to-noise-ratio (SNR) and the estimation accuracy was also studied. The proposed method based on correlation, approximates the Cramer-Rao Lower Bound (CRLB) in terms of the inverse mean square errors (MSEs) for different signal-to-noise ratios (SNRs). The method was tested by simulations to show the efficacy which yield result of variance which is theoretically equated to Cramer-Rao Lower Bound (CRLB). The result of the estimation was obtained with the help of construction of appropriate MATLAB code and valid when compared with other simulation results and related theory.

CHAPTER ONE

INTRODUCTION

1.1 Background to the Study

Time delay estimation (TDE) of signals has been field of great research interest in the recent time. It has several area of application which includes geophysics, radar, sonar, seismology, communication for detecting, identifying, localizing radiating source, ultrasonic etc. Subject to an area of application, time delay estimation can be classified into two types, namely, time of arrival (TOA) estimation and time difference of arrival (TDOA) estimation (Ehrenberg *et al.,* 1978; Owsley & Swope, 1981; Tremblay *et al.,* 1987; Wu *et al.,* 1999).

Before now, time delay estimation aim at measuring time delay between the transmission of a pulse signal and the response of its echo, which is often of main interest to an active system such as radar system, active sonar, subsequently, as the name implies, it pave way to determination of relative time difference of arrival between two spatially separated sensors which often the concern to a passive system such as microphone array systems, passive sonar. However, there is an essential connection amongst time of arrival (TOA) and time difference of arrival (TDOA) estimation, their fundamental difference is plainly philosophical. In the earlier aim, the "clean" reference signal, that is to say the transmitted signal is well known, to the extent that the time delay estimate can be determine easily based on a single sensor usually with the help of matched filter concept. On the contrary, in the latter, no such explicit reference is available, and the delay estimate is often acquired by comparing the received signals at two or more spatially separated sensors (Wu *et al*., 1999).

In this research work, the problem of time delay estimation for targets is reduced to that of parameter estimation associated with the signals received. The research work is concerned with the problems of detecting signals and / or estimating time delay as the parameter estimation which was based on time-frequency realizations of Wigner distribution (Ijima *et al*., 2002).

The research work aim at determining time delay of a signal via the time-frequency representation (TFR) which avail the energy concentration along the frequency axis at any given time. It can be achieve through the following objectives. Firstly choice of the signal was made to be linear frequency modulation (LFM) due it quality, after which the signal was represented in Time-frequency domain. However, the representation has its own drawback which makes the interpretation and estimation of the time delay difficult of which effort was put in place to overcome such defects to stabilize the signal and allowed the estimation of the delay with ease at different signal to noise ratio (SNR). After which it compare the performance of Wigner Ville distribution with other existing methods. Monte Carlo simulation was perform at various signal to noise ratio (SNR) and compare the results with the existing theory (Cramer-Rao-Lower-Bound (CRLB))

There is increasing interest in time-delay estimation of signals which are transmitted to a target and received with corrupting non-stationary random noise. It is unnecessary to say that the necessity for determining time delay arises in sensor array systems. The problem of time-delay estimation for targets reduces to that of estimating the parameter associated with the received signal. In this research work, the focus was given to the problems of detecting signals and/or estimating time-delay as the parameter estimation, and a concept was developed based on correlation function, and it was constructed using MATLAB

based on time-frequency realizations of Wigner-Ville Distribution (WVD) (Ijima *et al*., 2002; Ijima & Ohsumi, 2003).

Radio detection and ranging or RADAR is an important sensor for target detection and tracking. Radar in military perspective is usually used in tracking enemy missiles, ships, aircraft, and satellites. An effective radar system should be able to provide information about the position of enemy targets which may later be used in threat recognition and evaluation. Radar is also used extensively in civilian field applications such as air traffic control, ocean surveillance, terrestrial traffic control and weather sensing (William, 2013).

The ability to determine the direction of an emitted signal is of importance in a wide range of fields, including electronic warfare support (ES) and electronic intelligence (ELINT). For a digital receiver, direction finding is necessarily preceeded by signal detection, and parameter estimation, and this leads to the theory of waveform design. In developing the theory of waveform design and analysis, it is found that a simplification of the analysis can be achieved by working in the time-frequency domain (Rihaczek, 1996). For effectiveness and efficiency of this research work, linear frequency modulation (LFM) signal is chosen as the only signal of reference. Detecting and estimate LFM signals is an important tool for time delay estimation, the major problem of estimating time delay between signals received in the presence of noise have considerably drawn attention in different applications; such as in radar, communication, sonar, bioengineering, and so forth (Carter, 1993).

Signal representation in one domain has a deficiency in the frequency description of the signal. On the other hand the Fourier Transform (FT) of the signal cannot describe how the spectral content of the signal alters with respect to time. Therefore a time variable is induced in FT to obtain information about the changes of spectra content of a signal with respect to time (Cohen, 1995; Lusin & Agrez, 2011). Both frequency and time domain constitute alternative ways of looking at a signal. However, FT permits transformation from one domain to another, which means that, it does not permit the combination of the domains at the same time; most of time domain representations cannot be easily accessible in the alternative frequency domain (Hlawatsch & Boudreaux-Bartels, 1992).

The Fourier transform and its inverse (Hlawatsch & Boudreaux-Bartels, 1992) as mathematically shown in equation 1.1 and 1.2 respectively only establishes on one to one relation between time domain signal $x(t)$ and that of frequency domain $X(f)$.

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi tf} df \tag{1.1}$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi tf} dt \tag{1.2}$$

where $x(t)$ is the signal in time domain, $X(f)$ is signal in frequency domain, t is the time and f is the frequency.

Frequency domain and time domain establishes two different methods of viewing signal. However, FT permits a passage from one domain to another; it does not permit combination of the two domains at the same time. Particularly, most time information is not simply accessible in the domain of the frequency. While the spectrum $X(f)$ indicates

the strength with which any kind of frequency f is contained in the signal $x(t)$, which does not generally provide easy ways to interpret information of the localization time of the spectral components, which is what give rise to its deficiency as discussed (Hlawatsch & Boudreaux-Bartels, 1992).

A signal is said to be a description on how a parameter varies with another parameter or any physical quantity that varies with time, space or any other independent variable is said to be signal. Signals can also be classified as stationary and non-stationary. Stationary signal has properties of time-invariant that is to say the property does not change with time. One of the fundamental ways of studying a signal is time. However, there are other ways of representing signals, among all one of the most important is frequency. The analysis of non-stationary and time varying signals via time-frequency has been a great area of research interest.

Only linear frequency modulated (LFM) signals was considered for the accomplishment of the research aim which fall under the class of non-stationary signals. The analysis of non-stationary signals as per the chosen signal cannot accomplish by the simple use of classical time-domain representations or frequency domain representations because of their deficiency. For this kind of signals, the idea of Time-Frequency Distribution was initiated and used to formulate the interesting properties needed for a tool for Time-Frequency Signal Analysis. It has been proven that the method based on quadratic time-frequency distributions, that is to say Wigner-Ville Distribution is the most effective methods of analyzing signals of practical signal of interest (Shafi, 2009).

Haven selected the choice of signal to be used, the signal was presented in Wigner-Ville form which allowed us to analyze, interprets and determine the delay effectively.

Time-delay estimation is one of important issues in signal processing. Most of conventional methods have been developed by assuming the corrupting noise is stationary with time-invariant power spectrum.

1.1.1 Methods of TDE

For more than two decades, the problem of estimating time delay between signals received in the presence of noise has been considered for a variety of applications; such as in acoustics, radar communication, microphone array processing systems and speech recognition.

There are several algorithms to estimate the time delay (Carter, 1993). The cross-correlation (CC) method is one of the basic solutions of the TDE problem (Carter, 1993). Many other TDE methods develop based on this algorithm. The CC method cross-correlates the outputs of the received signals and consider the time argument that corresponds to the maximum peak in the output as the estimated time delay. To improve the peak detection and time delay estimation, various filters, or weighting functions, have been suggested to be used after the cross correlation (Knapp & Carter). The estimated delay is obtained by finding the time-lag that maximizes the cross-correlation between the filtered versions of the two received signals. Time-Delay estimation may face some practical problems, such as, room reverberation, acoustic background noise and the short observation interval. SNR plays an important role in TDE, an SNR threshold is considered as a distinguishable standard between the high and low SNR

There are various TDE methods that have advantages in easing hardware implementation, reducing computation difficulty, and increasing precision. Some of TDE methods are explain as follows:

1.1.1.1 Cross-correlation (CC) method

Computation of cross correlation function between received signals at two different microphones is one of the common methods of estimating time delay. Thereafter, locate the maximum peak at the output indicate the estimated time delay (Carter, 1993). The cross correlation can be modeled as:

$$R_{r1r2}(\tau) = E\left[r_1(t) r_2(t-\tau)\right] \tag{1.3}$$

$$D_{CC} = \underset{\tau}{arg\,max}\left[R_{r1r2}(\tau)\right] \tag{1.4}$$

where $r1\ and\ r2$ are the outputs of the received signals and D_{CC} is the cross-correlation

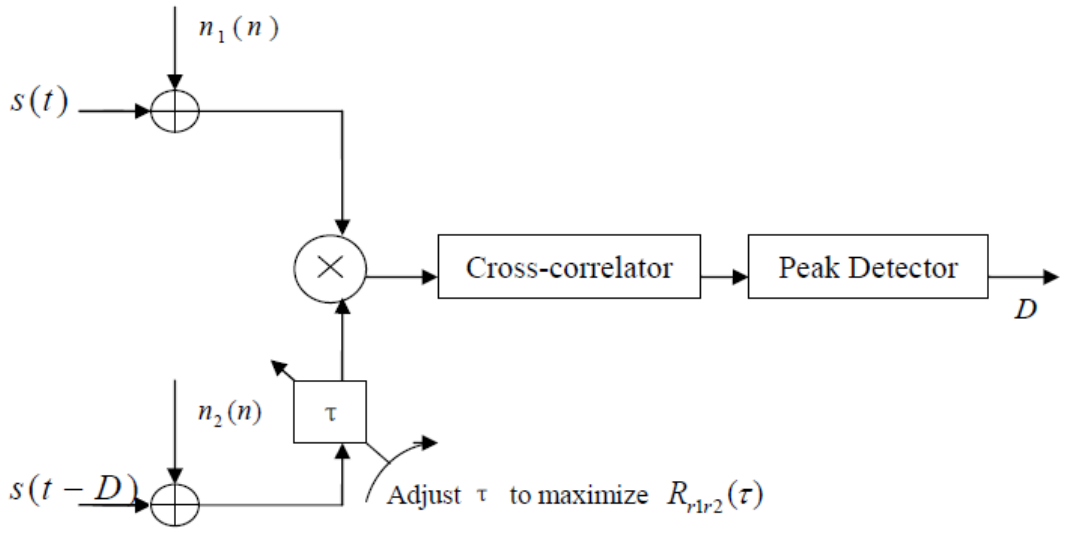


Figure 1.1: Cross-correlation Processor

1.1.1.2 Phase transform (PHAT) method

A way to sharpen the cross correlation peak is to whiten the input signals by using weighting function, which leads to what is called generalized cross-correlation technique (GCC). Figure 1.2 shows the block diagram of a generalized cross-correlation processor. The PHAT is a GCC procedure which has received considerable attention due to its ability to avoid causing spreading of the peak of the correlation function (Wang & Chu, 1997). It can be mathematically expressed as:

$$R_{r1r2}(\tau) = \int_{-\infty}^{\infty} \psi_p\,(f) G_{r1r2}(f)\, e^{j2\pi f\tau} df \tag{1.4}$$

$$\psi_p\,(f) = \frac{1}{|R_{r1r2}(\tau)|} \tag{1.5}$$

$$D_p = \arg\max[R_{r1r2}(\tau)] \tag{1.6}$$

where $G_{r1r2}(f)$ is the cross-spectrum of the signal received, $\psi_p\,(f)$ is the phase transform (PHAT) weighting function. Liang *et al.* (2002), state that only phase information will be preserved after cross-spectrum is divided by it magnitude. In an ideal condition, no noise is present; the processor approaches a delta function which is said to be centered at the correct delay.

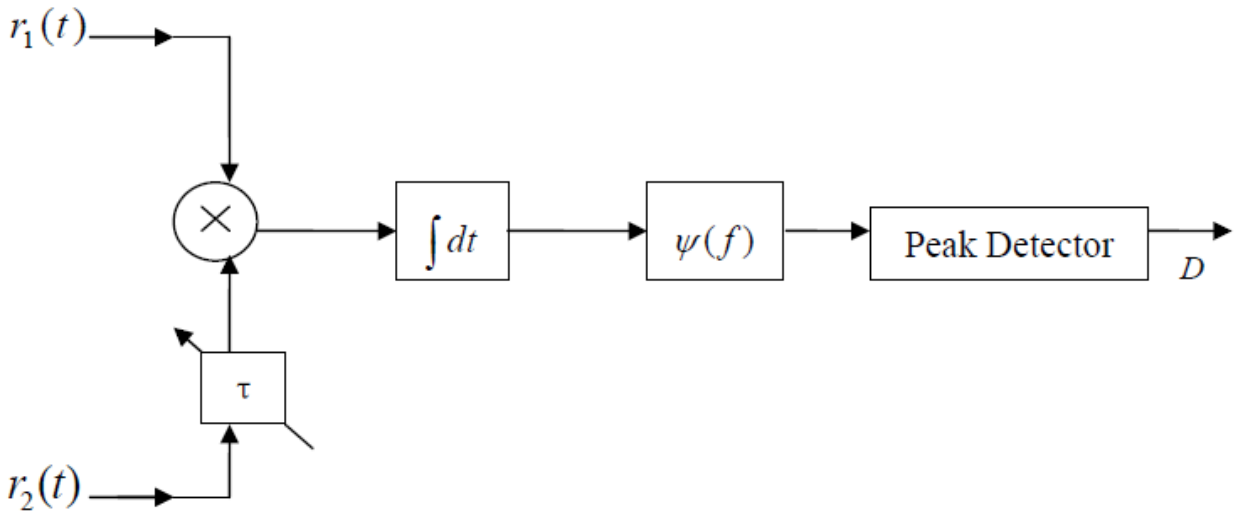


Figure 1.2: Generalized Cross-correlation Processor

1.1.1.2 Average square difference function (ASDF) method

The ASDF method is based on finding the position of the minimum error square between the two received noisy signals and considering this position value as the estimated time delay (Jacovitti & Scarano, 1993).

$$R_{ASDF} = \frac{1}{N}\sum_{n=0}^{N-1}[r_1(n) - r_2(n+\tau)]^2 \quad (1.7)$$

$$D_{ASDF} = \arg\min[R_{ASDF}(\tau)] \quad (1.8)$$

The performance of the ASDF estimaton is that it gives better estimation in a noise free environment while the direct correlation does not (Jacovitti & Scarano, 1993). Zheng and Tjeng (2003) highlighted that ASDF requires no multiplication, which is the most

significant practical advantage over the other methods. Also, this technique does not require the knowledge of the input spectra.

1.2 Statement of the Problem

Time representation of a one-dimensional signal lacks the frequency description of the signal. On the other hand, the Fourier Transform (FT) of the signal cannot describe how the spectral content of the signal alters with respect to time; as such there is need to represent the signal in two dimensions which enable effective determination of time delay. A non-stationary signal such as LFM signals whose spectral description depends on time is best analyzed with time-frequency distribution (TFD).

Time-Frequency representations (TFRs) are classified as Linear TFRs and Quadratic TFRs. Linear TFRs (short time Fourier Transform, Gabor Transform (GT), wavelet transform, etc.) obey the principle of superposition. Linear TFRs offer no cross-terms but have low resolution of auto-components. Quadratic TFRs (Wigner Distribution (WD), S-method, etc.) offer better resolution of auto-components but have cross-terms (Cohen, 1995; Boashash, 2003). There is no unique TFR that tackles all possible applications. The choice of a particular TFR depends on specific application at hand. However, TFRs have proved themselves to be the key to successful extraction and classification of signals in numerous applications. TFRs are often compared in terms of their ability to suppress cross-terms, resolution performance and mathematical properties (Shafi *et al.,* 2009).

The presence of the noise leads to false detection and inaccurate signal parameters estimation. To detect and estimate LFM signals is an important problem in time delay estimation between two received signal in a noise free and noisy environment. When the

signal is received at the receiver, it is delayed in time, attenuated, and contaminated by random noise (Ijima *et al.,* 2002).

1.3 Aim and Objectives

1.3.1 Aim

The aim of this research is to determine the time delay of a signal via the time-frequency representation (TFR) which helps to define the energy concentration along the frequency axis at any given time.

1.3.2 Objectives

The objectives of this research work are to:

(i) Obtain Time-frequency representation (TFR) of signals.

(ii) Reduce the cross-terms that makes the interpretation of the signals more difficult and estimate the time delay at different signal to noise ratio (SNR)

(iii) Compare the performance of Wigner Ville distribution with other existing methods

(iv) Perform Monte Carlo simulation at various signal to noise ratio (SNR) and compare the results with the existing theory (Cramer-Rao-Lower-Bound (CRLB))

1.4 Significance of the Study

Time-delay estimation of signals is an important issue in signal processing. Time-Frequency representation as a two-dimensional function provides simultaneously temporal and spectral information and therefore it helps in the analysis of non-stationary signals such as voice signals, biomedical signals, and radar signals. It helps to provide

information that is not available in either time or frequency representation. TFR helps to provide necessary information such as auto components (quadratic chirp, Gaussian atom and sinusoid etc.) in signal.

1.5 Scope

The scope of this research work time delay estimation of signal using Wigner-Ville distribution as the only means of time-frequency representation (TFR) which avail the energy concentration along the frequency that exist at any given time. It is also tie down to the following: only the linear frequency modulation (LFM) signal among other form of signals because of its quality characteristics, it provides information which is unavailable in time or frequency representation, have good range resolution and more Doppler tolerant than other form of signals, after which the signal was represented in Time-frequency domain. However, the representation has its own drawback which makes the interpretation and estimation of the time delay difficult of which parameter selection in MATLAB was put in place to overcome such defects to stabilize the signal and allowed the estimation of the delay with ease at different signal to noise ratio (SNR). After which the simulation result shows the signal representation in Wigner Ville distribution form of representation. Monte Carlo simulation was carried out at various signals to noise ratio (SNR) and the results was compared with the existing theory (Cramer-Rao-Lower-Bound (CRLB)).

CHAPTER TWO

LITERATURE REVIEW

2.1 Signals and Systems

2.1.1 Signals

Anything that carries some information is said to be signal. A signal is a description on how a parameter varies with another parameter more also any physical quantity that varies with time, space or any other independent variable is said to be signal. A system is an interconnection of components, it is a physical device that performs an operation on an input signal and produces another signal as output. Most examples of signals are Electrocardiogram (ECG) signal which helps in providing information about the health of a person's heart and Electroencephalogram (EEG) signal which provides information about brain activity of a person. Other examples are seismic signals, speech signal, voltage changing over time in an electronic circuit, or brightness varying with distance in an image. A signal can be a function of one or more independent variables. If a signal depends on only one variable, then it is known as one dimensional signal. AC power supply signal and speech signal are functions of one independent variable, namely time. On the other hand, if a signal depends on two independent variables, then the signal is known as two dimensional signals. For the purpose of this research work we will be concentrating on this kind of signal (Babu, 2011).

Naturally, signals vary greatly and in most cases it is very difficult to characterize them. Signals can be classified as deterministic or stochastic (random). A deterministic Signal is said to be clearly known while a stochastic Signal is one obtained from a collection of

signals which are characterized by properties like variance and expected value. Signals can also be classified as stationary or a non-stationary. A Stationary signal does not change with time and is said to be time-invariant (Babu, 2011).

One of the fundamental ways of studying a signal is time. However, there are other ways of representing signals one of the most important is frequency. The analysis of non-stationary and time varying signals via time-frequency has been a great area of research interest in recent times (Sandsten, 2013).

The common type of signal in analog electronics is potential difference (voltage) which varies with time. Both parameters can assume a continuous range of values and such signal is said to be continuous. In contrast, passing the signal through analog-to-digital converter forces each of the two parameters to be quantized. For instance, consider converting 12 bits at a sampling rate of 1000 samples per second. The voltage is shortened to 4096 (212) possible binary levels and the time defined at one millisecond increments. Signals formed from parameters that are quantized in this manner are said to be discrete signals or digitized signals. Mostly, continuous signals exist in nature, while discrete signals exist inside computers though there are exceptions. It is also possible to have signals where one parameter is continuous and the other is discrete. Since these mixed signals are quite uncommon, they do not have special names given to them, and the nature of the two parameters must be explicitly stated.

Signals can be categorized into two: Non-stationary and stationary signals. Non-stationary signals have frequency content, or statistical properties, that change over time, unlike stationary signals that have a constant frequency, or constant statistical proprieties, over time. Many signals in practice, such as those found in speech processing, biomedical applications, seismology, machine condition monitoring, radar, sonar, telecommunication, and many other applications are Non-stationary (Boashash, 2003).

Most of the signals that we encounter in engineering practice are not easily interpreted. Therefore, tools that can help process the information encoded in the signals are necessary. There are two classical signal representations which are time-domain representation, $x(t)$ and frequency representation, $X(f)$ (Boashash, 2003). However, both approaches are not suitable in signal's feature extraction. This is because of the lacks of frequency descriptions of the signals in time domain and the inability to depict the changes in spectral content of the signals over time in frequency representation. A more appropriate analysis for non-stationary signals is the joint time-frequency analysis (TFA) which is able to provide information about the frequency components and the temporal behavior of the signal. The main objective of TFA is to define the energy distribution along the frequency axis at each time instant.

Signals are used to represent information in formats such as video, images, speech, radio communications, radar, and may others (Cumming & Wong, 2005). Signals can either have a continuous or discrete-time variable representation (Ozdemir, 2013). A continuous-time signal, often referred to as an analog signal, is defined along a continuous real-valued time variable. Conversely, a discrete-time signal is defined at integer-valued discrete times (Ozdemir, 2013). In an actual radar system, the antenna

transmits a continuous-time signal. The signal must be converted to discrete-time in order to perform signal processing and analysis. Throughout this thesis, the signals referenced are continuous –time.

In the issue of signal estimation, the estimation of the frequency of Frequency Modulated (FM) signals. FM Signals are used in many engineering applications, such as radar, sonar, acoustic emission and communications (Boashash, 1992). Such signals contain the intended information in the frequency content. Let us consider, as an example, the application in telecommunications where an information bearing signal is sent through a communication channel. Depend on the real life condition of the communication channel. The transmitted FM signal is often attenuated and corrupted by different noise processes (e.g. additive noise, impulsive noise) and interference agents (other signals in the same communication channel or from other channels). At the receiving end, one needs to recover the intended information from the observed single having been corrupted: for the case of an FM signal in particular, that is the frequency content. Accurate frequency estimation leads to accurate recover of the true information. In many cases, it is assumed that the received FM signal has deterministic amplitude over the duration of observation. While this is a valid assumption in a wide range of scenarios, there are important applications under which this assumption is inappropriate: the received signal has instead, random amplitude. This phenomenon appears in applications such as radar imaging, weather radar, underwater acoustic applications, and wireless mobile communications. In these applications, the random amplitude modulation shows up when the medium is dispersive and/or when the assumption of point target is not valid (Dwyer, 1991; Haykin, 1994). In the literature, the original signal is often said to be corrupted by a Multiplicative

Noise process. Different underlying media result in different structures of the Multiplicative Noise. Anyhow, the recovery process of the information bearing frequency becomes more difficult. In the case of non-stationary signals, a time frequency approach using Higher Order Statistics (HOS) of TFD was proposed by Boashash (1993) and Boashash (1998) for real valued Multiplicative Noise. Many other approach for real valued model are seen in (Besson & Castanié, 1993; Morelande & Zoubir, 2002). In contrast, the complex valued Multiplicative Noise model has just been considered recently in Ghogho *et al*. (2001) and Azemi *et al*. (2002).

From the address problems above, the research work focus on a particular type of non-stationary signals. The Linear Frequency Modulation (LFM) signal. Though presenting the simplest form of FM non-stationary, LFM signals are both fundamental for research as well as practical for applications such as radar (Rihaeczek, 1985) and in communications (Amin, 1997; Gupta & Papandreou, 2001).

2.1.2 Linear frequency modulated (LFM) signals:

LFM signals commonly known as linear chirps are the most commonly used signals in radar systems as it can be easily generated, have good range resolution and more doppler tolerant. A linear FM chirp has a linear time frequency description as its frequency varies linearly over the pulse duration of the signal. In case of LFM frequency increases (up chirp) or decreases (down chirp) linearly with time.

2.2 Spectrum Analysis

The step in the analysis of modern spectrum (discrete-time, sampled) was introduced by Sir Arthur Schuster in 1898. As a result of his work, the invention of period-gram was made possible. The description of period-gram was made with the help of heterodyne technique where the low-pass filter is filter with impulse response which corresponds to a time limited rectangle sequence (window). Since the development of Fast Fourier Transform (FFT) algorithm in the year 1965 by James Cooley and John Tukey, the method has frequently been used for spectrum estimation.

FFT is considered to be the most effective and efficient channel of calculating Fourier transform of a signals which helps to discover the structure of Fourier algorithm to minimize summations and multiplications. As a result of that, it makes the Fourier transform to be the tool and not only theoretical description (Brillinger & Tukey, 2002).

To determine a spectrum that is reliable with best frequency resolution from a sequence of (of-ten short length) data in the recent time is an area of research interest. For stationary time-series, according to literature, there are two major approaches the windowed period-gram (and lag-windowed covariance estimates) which represents the non-parametric approach or the classical spectral analysis and the model based, usually an Auto-Regressive (AR), which represents the parametric approach or the modern spectral analysis. The combination of these two approaches and its development has been effectively utilize for over two decades now in quit some number of application (Stoica & Moses, 1997).

The analysis techniques of spectrum rely on the fact that the properties, for example the contents of the frequency of the signal changes slowly with time. If this is true, the time-frequency spectrum of the signal can be estimated and visualized.

2.3 Time-domain representation of Signal

Representation of signal in time domain has no information of its frequency description of the signal, on the way round the frequency representation of the signals lacks time contents of the signals. In view of this, the Fourier Transform (FT) of the signal cannot in any way describe how the frequency content of the signal alters with respect to time, the Fourier Transform (FT) and its invers can only establish a one to one relationship between the frequency domain representations (spectrum $X(f)$) and the time domain (signal$x(t)$) (Cohen, 1995; Lusin & Agrez, 2011).

2.4 Frequency-domain representation of Signal

The Fourier transform of any given continuous signal $x(t)$ can be define as (Hlawatsch & Boudreaux-Bartels, 1992)

$$X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \partial t \tag{2.1}$$

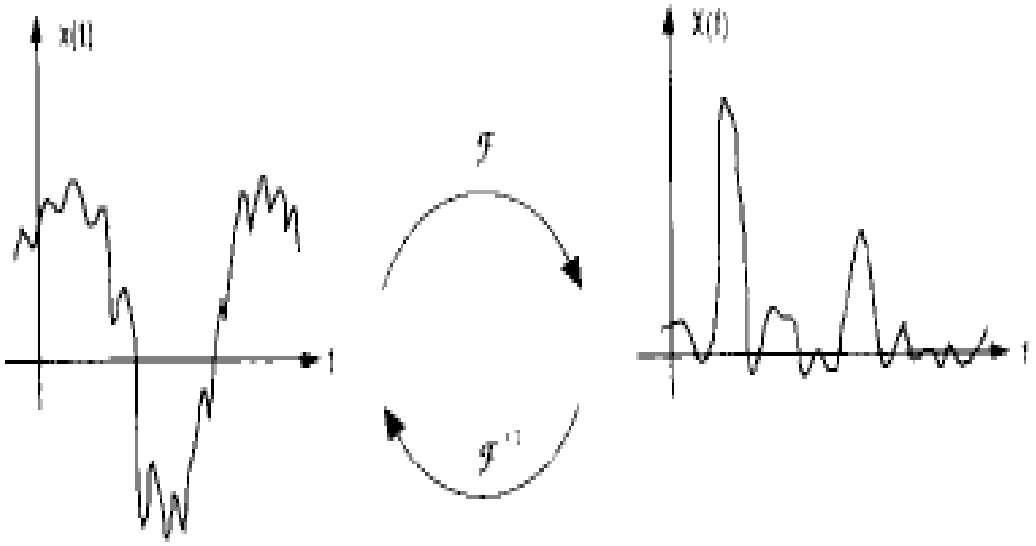
for the recovery of the signal $x(t)$in equation 2.1 inverse Fourier transform is employed given as:

$$x(t) = \mathcal{F}^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi ft} \partial f \tag{2.2}$$

Equations (2.1) and (2.2) are the mathematical expression that allows the transformation of one dimensional signal to the other (representation in time and frequency).

The frequency domain and the time domain form two different alternative ways of reading signal, however, the FT give room for transformation from one domain to the other, but it does not allow combination of the two domains. Particularly, most time information cannot be easily accessible in the frequency domain. While the spectrum $X(f)$ shows the overall strength with which any frequency f is contained in the

signal$x\ (t)$, which does not generally provide easy-to-interpret information about the time localization of spectral components. Figure 2.1 shows and illustration of the transformation of the signals.



$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi tf} df \qquad X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

Fig. 2.1 Time domain and frequency domain: dual approach to signal analysis. With the help Fourier transforms (Hlawatch & Boudreaux-Bartels, 1992).

2.5 Time-Frequency domain representation of signals

In imperative to offer more insight into the nature of non-stationery signals, a new field of engineering and science has emerged in MATLAB with the help of signal processing tool (Boashash & Sucic, 2002). The introduction of time-frequency representation has led to new tools to represent and characterize the True Varying (TV) contents of non-stationary signals using TFDs. The essential characteristic of time-frequency is that it comprises a set of signal processing methods techniques and algorithms in which the two natural variables, time and frequency are used concurrently (Boashash, 2003). This contradicts the traditional signal processing methods in which time and frequency variables are used exclusively and independently. It is envisaged to see how time-frequency can be applied into the two fundamental issues in signal processing (i) signal estimation (Boashash, 1992) and (ii) signal separation (Cardoso, 1998) of non-stationary signals in the context of wireless communication.

There are various methods of viewing signals. One common method of visualizing a signal is in the time domain. This representation often plots the signal value (commonly a voltage or current that represents some other measurement, such as temperature or strain) as a function of time. Another useful signal representation is the frequency-domain view of the signal. This representation is typically based on a variation of the Fourier Transform and commonly plots the magnitude or phase content of a signal as a function of frequency. By examining the frequency-domain view of a signal, you derive information about your signal that might not be immediately apparent from an examination of the time domain representation. Although frequency-domain

representations such as the power spectrum of a signal often show useful information, the representations don't show how the frequency content of a signal evolves over time

The main aim of time-frequency representations is to allow a better signal interpretation than the ones available from simple time or frequency representations. TFRs should not contradict other signal information such as the energy spectral density, instantaneous signal power, and overall signal energy. TFR is said to be a two-dimensional function that simultaneously provides time and frequency information which therefore can be used to analyses non-stationary signals. It provides better information which cannot be found in frequency or time representation alone (Cohen, 1995).

Time-Frequency representation provides effective tools for analyzing non-stationary signals, whose frequency content varies in time, as well as linear time varying systems. Time-Frequency representation is a natural extension of both the time domain and the frequency domain processing, that involves representing signals in a two-dimensional space, and so revels "complete" information about the signal. Such a representation is intended to provide a distribution of signal energy versus time and frequency simultaneously.

In the field of telecommunications, wireless communication is growing at an explosive rate, stimulated by a host of important emerging applications ranging from third-generation mobile telephony, wireless personal communications, and wireless subscriber loops, to radio and infrared indoor communications, nomadic computing and wireless tactical military communications (Haykin, 1994). Signal processing has been playing an important role in providing solutions to some key problems encountered in communications in general, and in wireless communications in particular (Haykin, 1994).

The magnitude spectrum (frequency representation) of a signal gives no indication as to how the frequency content of the signal changes with time, important information when one deals with FM signals. Signal processing tool being a natural extension of both the time domain and the frequency domain processing, preserves and reveals this information about the signal. Time-Frequency involves, and is intended to provide a distribution of signal energy versus both time and frequency. For this reason, the TF representation is commonly referred to as a TFD (Dwyer, 1991).

Time-frequency representations (TFRs) of signals project one-dimensional signal of time, $x(t)$, to a two-dimensional function of frequency and time (t, f). However most of TFRs are time varying spectral representations with time running along one axis and frequency along the other axis. The value of the TFR surface above the time-frequency plane indicates that the spectral components are present at specific time.

Time-frequency representations are always used to modify, analyze and synthesize time-varying or non-stationary signals. Three-dimensional plots of Time-frequency representations surfaces have been used as pictorial representations enabling a signal processor to analyze how spectral components of a signal or system vary with time. Time-frequency representations inversion or synthesis algorithms is employed to recover a signal from a TFR model, thus allowing a time-frequency implementation of signal design, time-varying filtering, noise suppression, time wrapping, etc.

2.6 Classification of Time-Frequency Representations

Time frequency representations are classified into Linear TFRs, bilinear (Quadratic) TFRs and high-order species. Linear TFRs includes short time Fourier Transform, Gabor Transform (GT), wavelet transform, etc., which conform to principle of superposition. Linear TFRs has no cross-terms but the resolutions of auto signal components are low. Quadratic TFRs includes Wigner Ville Distribution (WVD), S-method, etc., which gives better resolution of auto-components but have cross-terms (Cohen, 1995; Boashash, 2003). The high order TF distribution can achieve higher concentration and special features. However, high order TF has relatively complicated computation (Zou *et al.*, 2016). Based on the classification, it can be deduced that there is no unique TFR that solves all possible applications. The choice of actual TFR depends on precise application at hand. Though, TFRs have proved themselves to be the key to effective extraction and classification of signals in various applications. TFRs are often compared in terms of their ability to suppress cross-terms, resolution performance and mathematical properties (Shafi *et al.,* 2009). Based on the existing literature, it has been proven that Wigner Ville Distribution (WVD) is the most effective quadratic TFR and it indicates it high resolution property for analysis of linear chirp signal and Gaussian atom, for the case quadratic component and multiple components Wigner-Ville Distribution has a cross terms problem (Boashash, 2003; Gröchenig, 2001). It has agreeable mathematical properties of WVD like infinite time and frequency resolutions, no window and preservation of frequency and time support and many more, the highlighted characteristics made WD more dominant tool for better analysis of signals (Classen & Mecklenbrauker, 1980; Lergal *et al,* 2011). In the similar manner, WD has drawback of its quadratic nature and brings about the cross terms which complicate the visualization of the frequency-time

plane (Hlawatsch, & Flandrin, 1997). Researchers have suggested various alternatives of WVD proposed to eliminate its cross-terms (Qian, 2002; Hlawatsch *et al.*, 1995).

2.7 Time Frequency Representation Feature Estimation Approaches

In evaluating a signal, one of the simplest signal processing procedures in the TF domain is using energy concentration. The higher the energy concentration in the TFR, it is more likely to produce more accurate result in feature extraction and classification. As reported in the literature, energy concentration in TF domain as feature extraction can be classified according its feature extraction approaches. The feature extraction approaches are given as follow:

1. TFRs based on signal decompositions (Stockwell *et al.*, 1996; Pinnegar & Mansinha, 2004).
2. TFRs based on Cohen's class (Boashash, 2016).
3. Rotated TFRs (Bastiaans *et al.*, 2002).
4. Signal optimized TFRs (Jones & Baraniuk, 1995; Chee *et al.*, 2014)

All of the feature extraction approaches mentioned above has their own unique characteristics. In some applications one approach might be better than the others. It all depends on the specific applications at hand.

2.7.1 Signal Decomposition Based TFRs

One of the advantages of signal decomposition based on TFRs is that they do not have cross-terms problems. The methods for decomposition often used to describe energy concentration such as short-time Fourier transforms (STFT) because of its implementation is simple. The wavelet transform (WT) and STFT are the examples of earlier TFRs based on the signal decomposition. The signal representations of STFT

whose basis or frame vectors fall under frequency translation group while WT is under dilations translation group (Wilson *et al*., 1992). Some other example of decomposition method is S-transform (Stockwell *et al*., 1996), short-time harmonic transform (STHRT) (Zhang *et al*., 2001) and short-time Hartley transform (STHT) (Pinnegar & Mansinha, 2004). The S-transform produces a higher energy concentration in the representation than the STFT due to the fact that the window is wider at lower frequencies. However, for the case of linear FM signal the STFT produce higher energy concentration than the S-transform. Additional advantages and shortcomings of TFRs is described in (Sejdi & Djurovi, 2009).

2.7.2 Cohen's Class based TFR

The main advantage and the feature of this representation and extraction is that Cohen's TFRs can produce significantly high resolution given that the inner interferences and cross-terms are kept at minimum. Most importantly, the work in this area focused on minimizing the cross-terms and inner interferences. In the presence of cross-terms, the representation of the signal in TF domain becomes ambiguous which lead to error in feature classification and extraction. The consequence of the cross-terms can be reduced by introducing a kernel function. The location of the auto-terms and cross-terms are best described in the ambiguity domain. For any signal, the auto-terms of the signal are located along and around the ambiguity axis which occurs maximally at the origin. However, the cross-terms are located further away from the axis (Boashash, 2016).

The Cohen's class can be expressed as (Sejdi & Djurovi, 2009).

$$TFD_x(t,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(v,\tau)G(v,\tau)e^{-j2vt}e^{-j\tau\omega}d\tau dv \qquad (2.3)$$

where G(v,τ) is the kernel function and A(v,τ) is the ambiguity function. Some of the kernel functions is introduced in (Williams & Jeong, 1992) under the framework of reduced interference distribution (RID). The examples of distributions that follows the idea of the RID class are Born-Jordan distribution, Choi-Williams distribution, Zhang-Sato distribution, Radial-Butterworth distribution, and hyperbolic distribution. It is important to mention that not every kernel can produce good signal representation in all applications. Some kernels are only suitable for certain class of signals such as in (Jones & Baraniuk, 1995) that introduced a kernel which only produce satisfactory result if the IF signal is a linear function. In dealing with higher order (more than second) of IF variations, the cross-terms issue is getting more profound. Though, the proposed L-class distribution is a better solution compared to Cohen's class distribution in the context of signal feature extraction (Sejdi & Djurovi, 2009; Signal & Letitrs, 1996)

2.7.3 Rotation of the TF plane

Rotation of the TF plane based TFRs involve the kernel rotation operation on the time-frequency distribution. The objective of rotating the TF plane is to improve energy concentration of the signals by matching the signal components with time or frequency axis (Bastiaans *et al*., 2002). For instance, in STFT the signal representation is free from the cross-terms interference but its energy concentration for each signal components is fairly lower than Wigner Distribution (WD). On the other hand, the energy concentration of each signal components in WD is higher than in STFT but greatly suffer from the

effect of the cross-terms. Introducing WD with a kernel function manages to minimize the interference caused by the cross-terms but it is limited to certain class of the signal. Thus, by rotating the TF plane and the kernel function to certain angle is proven to enhance the energy concentration in STFT and diminished the cross-terms in weighted WD (Sejdi & Djurovi, 2009).

There are various ways to achieved TFRs based on rotation of the TF plane such as fractional Fourier transform (FRFT), local polynomial Fourier transform (LPFT) and Radon-Wigner distribution (RWD). It is important to mention here that rotating a kernel that depend only on the product of lag and frequency delay did not show clear improvement in the cross-terms reduction for example the Born-Jordan, and the Choi-Williams (Bastiaans *et al*., 2002). One of the drawback of this approach is additional computational requirement in estimating the angle of the rotation and the example of LPFT integration in Wigner distribution that is able to produce very high signal concentration at the instantaneous frequency with a reduced problem dimension can be found in (Stankovic, 1997).

2.7.4 Signal Dependent TFRs

The feature extractors mentioned earlier provides solutions for improved energy concentration of TFRs. Some of them deals with reduce the spectral leakage issue, attenuating the cross-terms energy and aligning the analysis axis with the principal axis of the signal components. However, major drawback from fixed mapping is that they only produce satisfactory TFR for a certain class of signals only. Thus, it is important to have signal dependent representation which able to preserve the maximum concentration

of the signal components to its proper support in the TF domain (Sejdi & Djurovi, 2009). There are three available signal dependent TFRs approaches in the literature. These representations are in the form of concentration measures, reassignment method and signal optimized kernels/windows.

The concentration measures based TFR as proposed using STFT as a base function for signal concentration yield a fully automated determination of the optimal basis parameters. The basic idea of concentration measure is to find which parameter that yields the highest energy concentration of the signal in TF domain. Other researchers also have proposed different algorithm that distinguish between low and high concentrated components within the same distribution, it was proposed considering the use of Shannon of Rényi information measure to provide information on TFDs, Boahsash (2002) also proposed a resolution performance measure which was able to provide a high resolution TFRs for multicomponent signals.

Reassignment methods of TFRs are proposed initially for a spectrogram and later on generalized for other TF methods (Auger *et al.,* 1994). In this method, every TF point is recalculated around its centre of gravity and relocates to the better localization of the signal components. The disadvantage of this approach is computationally intensive and very sensitive to noise (Chassande-Mottin *et al.,* 1996).

Another approach for developing signal dependent TFRs is through signal optimized kernels/windows. One of the earliest methods of this stream of research is radially Gaussian distributions. The optimization problem is solved by limiting the signal dependent kernel to the class of radially Gaussian kernel and limiting the volume of the kernel to suppress the cross-terms. Some improvements are made later to radially

Gaussian distribution for improved computational complexity and compatibility in analysing very long signal and signals that changes its characteristics over time.

Adaptive cone-kernel quadratic TFR proposed in (Czerwinski & Jones, 1995) has many good characteristics in terms of resolution, cross-terms suppression, global time-support and high computational efficiency. The cone-length parameter is adjusted over time to match the local signal structure. Another example of signal optimized kernel is extended modified B-distribution (EMBD). The EMBD is the improved version of modified B-distribution (MBD) which attends the low pass filter in both lag-independent and Doppler-independent kernel with different parameters. The EMBD is more superior compared to MBD in terms of cross-terms suppression (Boashash & Ben-Jabeur, 2012).

2.8 Instantaneous Frequency (IF) Estimation

Practical signals show spectrum that differs over time. The theory of IF was introduced as an effort of learning how the signal components exist and changes with time. On the other hand, signal parameters such as frequency agility, pulse repetition period (PRP), modulation type and signal bandwidth (BW) can be easily determined given that the IF of the signal is accurately estimated. The techniques for IF estimation can be classed into few categories: zero crossing, adaptive IF estimation, moments of TFDs, peak of TFDs and time-varying autoregressive model (Boashash, 1992).

For a sinusoidal signal, the frequency of the signal is given as the inverse of its period or in the context of zero crossing, exactly half of the inverse of the interval of between zero-crossing. Zero-crossing implies the time when the signal crossing the axis in the graph of the function. Note that if the interval between the zero-crossing is not an integer number

of sample it will introduce new problem, quantization noise. In another research, it was proposed that taking the average number of zero-crossing (ZC) within certain window length could reduce the variance of IF estimation (Rabiner & Schafer, 1978). In another research, an adaptive window ZC-IF estimator was proposed to improve the accuracy of the IF estimation (Sekhar, 2004). The integration of adaptive window in ZC-IF estimator is reported to improve the IF estimation as much as 5dB in terms of instantaneous squared error compared to fixed window ZC-IF estimator.

In adaptive IF estimation, several different algorithms based on different approached wass proposed in the literature including modelling the data as a linear predictive process. From this approach, emerge two type of estimation algorithm: least mean square (LMS) algorithm and recursive least square (RLS) algorithm. The peak of the linear prediction based spectrum is used to estimate the IF (Griffiths, 1975). A lot of improvements have been reported on these two algorithms especially in reducing its computational complexity. However, the IF estimator based on linear predictive model is not suitable for rapid IF changes or in noisy environment (Boashash, 1992). Instantaneous frequency estimation can be derived from the first moment of the TFD. The IF law of a linear FM obtained from the first moment of WVD combined together with time-frequency filtering is fairly accurate (Boashash, 1992). However, estimating the IF using this technique is not favourable since its demand is high computational complexity from TFD, filtering and first moment calculation. Recent work shows that the concept of moments of TFD in discriminating the seizure from the non-seizure new-born.

Estimating the IF using peak of TFDs initially proposed for STFT and the performance of the estimation is well given that the signal is quasi-stationary. However, the limitation of the spectrogram lies on the fact that its optimum window is the reciprocal of the square root of the frequency rate, the estimation accuracy for rapidly slowing FM signals is very poor (Boashash, 1992). Rao and Taylor (1990) proved that peak based IF estimator from WVD is optimal for linear FM signal at high SNR. The IF estimation using generalized WVD (GWVD) later was proposed which is unbiased and its variance meet the CRLB at high SNR (Boashash & Frazer, 1992).. There are other work that are reported to apply the peak TFDs as the IF estimator.

The TFDs are normally used in the development of non-parametric IF estimator. Thus, it is worth to mention here that such IF estimator must consider following problems in order to produce accurate estimator: bias introduced by non-linearity in the IF function, cross-terms, and small and high noise influence and discretization error (Djurovic, 2011).

2.9 Noise

Noise can be characterized as any disturbance that tends to obscure a desired signal. Noise can be generated within a circuit or picked up from external natural or artificial sources. When noise is generated within a circuit, it is called intrinsic noise. When noise is picked up from an external source, it is called extrinsic noise Interference is noise that tends to obscure the useful signal. It is usually caused by electrical sources but can be induced from other physical sources such as mechanical vibration, acoustical feedback, or electrochemical sources. In addition to characterizing noise by its source, it is useful to

distinguish noise by its frequency spectrum and amplitude distribution. When noise power has a flat frequency distribution, it is called white noise (from the analogy of white light containing all frequencies). If you double the bandwidth of a system, you double the white-noise power. Noise that is inversely proportional to frequency is called 1/f, or pink, noise. Pink noise is present in many physical systems but is particularly important in low-frequency systems. In electrical systems, pink noise is generated when a current flows through a non-homogeneous material, such as in a carboncomposition resistor (a mixture of carbon and other semi-conductive materials). Pink noise is also generated in switches and other contact surfaces that are composed of dissimilar metals; it is then referred to as contact noise (Proakis, 1995).

The random nature of noise in time domain will, on occasions, causes a transmitted symbol to be distorted such that the receiver interprets it as a different symbol in the modulation scheme alphabet. Under these circumstances, a given average level of AWGN introduces an average number of symbol errors where each symbol error causes one or more bit errors at output of the receiver. The errors may be characterized by a bit error rate (BER) (Proakis, 1995).

2.9.1 Noise Measurement

Noise is unavoidable. Proper shielding and design can obtain the lowest noise level, but there will always be intrinsic noise in electronic systems, even if the power source is turned off. In interpreting and measuring noise, it is necessary to take into account the bandwidths involved and the type of noise. White noise, in theory, is infinite in bandwidth, having an even distribution from dc to infinity. This infinite bandwidth implies infinite noise power when applied to a resistor. In practice, electronic systems are

bandwidth-limited, so the noise is not infinite in energy. An important aspect of measuring noise is that the noise energy will increase as the bandwidth of the measurement system increases. For example, two meters may obtain different results when measuring noise in V rms on the same circuit because the bandwidth of the two meters may differ; the measurement equipment must have a bandwidth larger than the system being measured. Often filters are used to limit the bandwidth of the noise. For example, noise above 20 kHz is inaudible and does not diminish the signal quality in an audio system; therefore the measurement system is filtered above 20 kHz. The type of noise can affect the way noise is measured and calculated. For example, white noise, in the time domain, has a Gaussian voltage or power distribution about a dc level (the dc level is the signal level at any given period of time) and is best measured in terms of rms values of volts or power. Rms measurements are useful because they disregard the polarity of the noise. However, if the noise is an impulse (spike), then the peak or peak-to-peak value may have more meaning. For example, ignition noise may add very little to the rms value, but the effects can be very noticeable over a radio receiver. A look at the noise, using an oscilloscope and/or a spectrum analyzer, may be needed to determine the significant spectrum and the type of noise so the proper method of measuring the noise may be used (Proakis, 1995).

2.9.2 Signal To Noise Ratio (SNR)

Signal-to-Noise (SNR) is probably the most common and well understood performance measure characteristic of a digital communication system. Most often this is measured at the output of the receiver and is thus directly related to the data detection process itself.

Signal-to-noise ratio is a term for the power ratio between a signal (meaningful information) and the background noise: (Proakis, 1995).

$$SNR = \frac{P_S}{P_N} = 20 * \log\left(\frac{A_S}{A_n}\right)^2 \quad (2.4)$$

were P_S Ps is average signal power and P_n is average noise power, and A is root mean square (RMS) amplitude for signal and noise (for example, typically, RMS voltage). Both signal and noise power (or amplitude) must be measured at the same or equivalent points in a system, and within the same system bandwidth.

Because many signals have a very wide dynamic range, SNRs are usually expressed in terms of the logarithmic decibel scale. In decibels, the SNR is, by definition, 10 times the logarithm of the power ratio. If the signal and the noise is measured across the same impedance then the SNR can be obtained by calculating 20 times the base-10 logarithm of the amplitude ratio:

$$SNR = \frac{P_S}{P_N} = \left(\frac{A_S}{A_n}\right)^2 \quad (2.5)$$

The effects of noise on a signal are best analyzed as a ratio of the signal compared to the noise. This ratio is called the signal-to-noise ratio (S/N) and is often expressed in decibels. The noise is often referred to as the noise floor because its level stays fixed as the signal is varied in amplitude above it. There are three basic ways to improve the S/N ratio: (1) increase the signal strength, (2) decrease the noise level, and (3) limit the noise bandwidth. The larger S/N, the less the noise will affect the signal. If reducing the noise is not possible or practical or if increasing the signal will cause distortions elsewhere, then reducing the noise bandwidth may be the only approach to improving S/N. Although signal-to-noise ratio is normally expressed as a power ratio, it can be expressed as a

voltage ratio. If voltage ratios are used, then care must be taken to retain a constant impedance at the point of measurement. Measuring the noise with one instrument and measuring the signal with another instrument may cause errors if the loading effects on the circuit by the instruments are not the same (Proakis, 1995).

2.9.3 Additive White Gaussian Noise (AWGN)

A basic and generally accepted model for noise in communication channels, engineering, etc., is the set of assumptions that; the noise is additive, i.e., the received signal equals the transmit signal plus some noise, where the noise is statistically independent of the signal, the noise is white, i.e, the power spectral density is flat, so the autocorrelation of the noise in time domain is zero for any non-zero time offset, that the noise samples have a Gaussian distribution.

Mostly it is also assumed that the channel is Linear and Time Invariant. The most basic results further assume that it is also frequency non-selective ((Proakis, 1995).

Additive White Gaussian Noise (AWGN) is the statistically random radio noise characterized by a wide frequency range with regards to a signal in a communications channel.

2.10 Windowing

Although performing an FFT on a signal can provide great insight, it is important to know the limitations of the FFT and how to improve the signal clarity using windowing.

When you use the FFT to measure the frequency component of a signal, you are basing the analysis on a finite set of data. The actual FFT transform assumes that it is a finite data set, a continuous spectrum that is one period of a periodic signal. For the FFT, both the time domain and the frequency domain are circular topologies, so the two endpoints

of the time waveform are interpreted as though they were connected together. When the measured signal is periodic and an integer number of periods fill the acquisition time interval, the FFT turns out fine as it matches this assumption.

However, many times, the measured signal isn't an integer number of periods. Therefore, the finiteness of the measured signal may result in a truncated waveform with different characteristics from the original continuous-time signal, and the finiteness can introduce sharp transition changes into the measured signal. The sharp transitions are discontinuities. When the number of periods in the acquisition is not an integer, the endpoints are discontinuous. These artificial discontinuities show up in the FFT as high-frequency components not present in the original signal. These frequencies can be much higher than the Nyquist frequency and are aliased between 0 and half of your sampling rate. The spectrum you get by using a FFT, therefore, is not the actual spectrum of the original signal, but a smeared version. It appears as if energy at one frequency leaks into other frequencies. This phenomenon is known as spectral leakage, which causes the fine spectral lines to spread into wider signals.

We can minimize the effects of performing an FFT over a non-integer number of cycles by using a technique called windowing. Windowing reduces the amplitude of the discontinuities at the boundaries of each finite sequence acquired by the digitizer. Windowing consists of multiplying the time record by a finite-length window with amplitude that varies smoothly and gradually toward zero at the edges. This makes the endpoints of the waveform meet and, therefore, results in a continuous waveform without sharp transitions. This technique is also referred to as applying a window.

2.10.1 Windowing Functions

There are several different types of window functions that you can apply depending on the signal. To understand how a given window affects the frequency spectrum, you need to understand more about the frequency characteristics of windows.

An actual plot of a window shows that the frequency characteristic of a window is a continuous spectrum with a main lobe and several side lobes. The main lobe is centered at each frequency component of the time-domain signal, and the side lobes approach zero. The height of the side lobes indicates the affect the windowing function has on frequencies around main lobes. The side lobe response of a strong sinusoidal signal can overpower the main lobe response of a nearby weak sinusoidal signal. Typically, lower side lobes reduce leakage in the measured FFT but increase the bandwidth of the major lobe. The side lobe roll-off rate is the asymptotic decay rate of the side lobe peaks. By increasing the side lobe roll-off rate, you can reduce spectral leakage.

Selecting a window function is not a simple task. Each window function has its own characteristics and suitability for different applications. To choose a window function, you must estimate the frequency content of the signal.

a. If the signal contains strong interfering frequency components distant from the frequency of interest, choose a smoothing window with a high side lobe roll-off rate.
b. If the signal contains strong interfering signals near the frequency of interest, choose a window function with a low maximum side lobe level.

c. If the frequency of interest contains two or more signals very near to each other, spectral resolution is important. In this case, it is best to choose a smoothing window with a very narrow main lobe.
d. If the amplitude accuracy of a single frequency component is more important than the exact location of the component in a given frequency bin, choose a window with a wide main lobe.
e. If the signal spectrum is rather flat or broadband in frequency content, use the uniform window, or no window.
f. In general, the Hanning (Hann) window is satisfactory in 95 percent of cases. It has good frequency resolution and reduced spectral leakage. If you do not know the nature of the signal but you want to apply a smoothing window, start with the Hann window.

Even if you use no window, the signal is convolved with a rectangular-shaped window of uniform height, by the nature of taking a snapshot in time of the input signal and working with a discrete signal. This convolution has a sine function characteristic spectrum. For this reason, no window is often called the uniform or rectangular window because there is still a windowing effect.

The Hamming and Hann window functions both have a sinusoidal shape. Both windows result in a wide peak but low side lobes. However, the Hann window touches zero at both ends eliminating all discontinuity. The Hamming window doesn't quite reach zero and thus still has a slight discontinuity in the signal. Because of this difference, the Hamming window does a better job of cancelling the nearest side lobe but a poorer job of canceling any others.

These window functions are useful for noise measurements where better frequency resolution than some of the other windows is wanted but moderate side lobes do not present a problem.

The Blackman-Harris window is similar to Hamming and Hann windows. The resulting spectrum has a wide peak, but good side lobe compression. There are two main types of this window. The 4-term Blackman-Harris is a good general-purpose window, having side lobe rejection in the high 90s dB and a moderately wide main lobe. The 7-term Blackman-Harris window function has all the dynamic range you should ever need, but it comes with a wide main lobe.

A Kaiser-Bessel window strikes a balance among the various conflicting goals of amplitude accuracy, side lobe distance, and side lobe height. It compares roughly to the Blackman-Harris window functions, but for the same main lobe width, the near side lobes tend to be higher, but the further outside lobes are lower. Choosing this window often reveals signals close to the noise floor.

The flat top window is sinusoidal as well, but it actually crosses the zero line. This causes a much broader peak in the frequency domain, which is closer to the true amplitude of the signal than with other windows.

2.11 Other Related Review

Ran Tao *et al*. (2009) in his paper titled “time delay estimation of chirp signals in the fractional Fourier domain, aimed at obtaining the delay on the coordinates that the signal is corrupted at the time signal was received via the target. The conventional time delay estimation methods are based on cross correlation or matched filtering which literature

shows that is the best and most useful algorithm (Knapp & Curter, 1976; Nandi, 1995; Lin, 1981).

Sharma and Josh (2007), proposed various time delay estimation methods using fractional furrier transform to improve performance of the time delay, however, in the course of their result presentation, and analysis, consideration was not put in place to checkmate mathematical expressions of the results (output) of signal to noise ratio (SNR) and also efficiencies of time delay estimation related to furrier transformation (FRFT), of resent, the FRFT has been given much attention in many areas of research such as signal processing, quantum physics, optical system (Almeida, 1994). The operation shows the notation of signal $f(t)$ having an angle α in time frequency plane. The fractional furrier transform of a signal $f(t)$with an angle α can be defined as (Almeida, 1994).

$$F_\alpha = \int_{-\infty}^{+\infty} f(t) k_\alpha(u,t) dt$$

$$= \begin{cases} A_\alpha \int_{-\infty}^{+\infty} f(t) e^{\frac{u^2+t^2}{2} cot - jut\ cos\alpha\ dt} & \alpha \neq 2n\pi, 2n\pi + \pi \\ f(t) & \alpha = 2n\pi \\ f(-t) & \alpha = 2n\pi + \pi \end{cases}$$

where $k\ (u,t)$ is the transformation kernel, A=$\sqrt{(1 - \frac{jcot\alpha}{2\pi}}$ and n an integer. The FRFT reduces to the conventional furrier transform (FT) where, $\alpha = 2n\pi + \frac{\pi}{2}$ that is F$f_{2n\pi+\frac{\pi}{2}}(U) = F(U)$ where $F(u)$ denotes the FT.

Contrary to the conventional Fourier transform (FT), property of delay of the FRFT might have different behavior with respect to the location of the extreme FRF spectrum. Consequently, the TDE is converted to problem of estimating the parameter in the fractional furrier domain (FRFD). In various applications, usually chip signals are mostly preferred.

Ran Tao et al, in their paper titled "time-delay estimation of chip signals in the fractional Fourier domain" adopted fractional Fourier transform method of delay estimate reason been that FRFT has better concentration of chips signals. The time delay estimation of signal was achieved by applying the delay property of FRFT. In their paper they presented FRFT based time delay estimation method which needs less calculation and is also different from correlation based method as presented by (Pei, 2000; Sharma & Josh, 2007). In their results, they also present a result of statistical analysis in terms of signal to noise ratio (SNR) as well as the accuracy of the estimation which is related to theory of Cramer-Rao lower bound (CRLB) for the time delay estimation. The simulation results of their work were given to attest the validity of the proposed estimation method.

From their review, it was shown that FRFT computation based methods does not take much longer in comparison to conventional FT (Ozaktas *et al*., 1996), their proposed method can be less computational costly in implementation. If the estimation of the time delay is under noise, free, then the system is efficient, which in real sense is not possible, however, if the estimation of the delay is under noisy environment, it tends to give what we know as estimation error which does not give accurate results of the delay estimation which can be misleading. It was shown in their result that the expression has the same form of Crammer-Rao-Lower bound theory in the estimation of time-delay, which

indicates that the variance of TDE is related with bandwidth and energy of the chip signal, similarly the variance of noise i.e. the variance of the proposed TDE is equal to the theory CRLB which confirm the effectiveness of their proposed TDE method.

In their results, it was indicated that they evaluated the performance of fractional time delay estimation selecting a chip signal with uniform amplitude with modulated rate of -4 x 10^{10} having a bandwidth of about 3.6276×10^{6}. The Monte Carlo simulation runs, it was also shown that the simulation was carried out on free noise environs and not a free noise environs. More also, the result show that the SNR is increasing, the variance approaches the CRLB as shown in Figure 2.2.

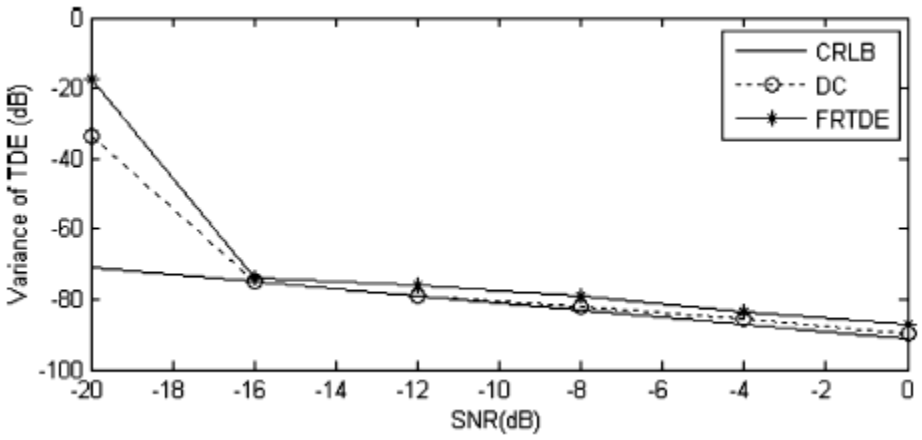


Figure 2.2 Variance of time-delay estimation as a function of SNR.

Gradient methods of time delay estimation are based on updating the delay by a vector that depends on information about the cost function to be minimized. The gradient algorithms normally involve expanding the cost function as a second order Taylor's expansion around the estimated delay. The choice of gradient algorithm for an application depends on the desired speed of tracking and the computational resources available. It is important that the error surface in the direction of the delay should be unimodal if a gradient algorithm is to be used successfully. However, the error surface is often multimodal. In these circumstances, strategies for locating global minima may involve multiple optimization runs, each initiated at a different starting point with the starting points selected by sampling from a uniform distribution. The global minimum is then the local minimum with the lowest cost function value among all the local minima identified. Gradient algorithms based on the Gauss-Newton, steepest descent and least mean squares (LMS) method has been defined. Alternatively, the cross-correlation of two signals may be used to estimate the time delay between the two signals, as the time at which the cross-correlation term is maximized corresponds to the time delay estimate. Most cross-correlation methods, and variations of the method, are off-line in nature though some approaches have on-line potential. Other authors use the technique to estimate time delays in MIMO environments or between multiple sensors and multiple targets. Other authors also use power spectral density techniques to calculate the time delay. The time delay may also be determined from a higher order spectral approach though the use of these techniques are suited to a restrictive range of problems, in which noise signals on the input and output to the process cannot be effectively dealt with by pre-processing. Finally, other off-line delay estimation algorithms have been defined; one

example is discussed by Kinetic in which the time delay between two sensors may be found by determining the maximum of the probability density function of the delay from a given prior distribution. Other off-line time delay estimation algorithms are also discussed. On-line implementation of non-cross correlation based algorithms has also received attention; for example, investigate the performance of a simulated annealing algorithm in the estimation of the amplitude scaling factors and the time delays of the separate arrivals in a signal composed of closely spaced arrivals with added noise. The method is particularly interesting as the cost function to be minimized has local minima that make the application of calculus based minimization techniques (such as the gradient algorithms) difficult; the simulated annealing algorithm has the ability to slide through local minima.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Material

MATLAB software is the material employed in the conduct of this research work.

3.1.1 Overview of the MATLAB Environment

The MATLAB® high-performance language for technical computing integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses include: Math and computation, algorithm development, data acquisition, modeling, simulation, and prototyping, data analysis, exploration, and visualization, scientific and engineering graphics application development, including graphical user interface building (Brian, 2001).

MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. It allows you to solve many technical computing problems, especially those with matrix and vector formulations, in a fraction of the time it would take to write a program in a scalar non-interactive language such as C or FORTRAN.

The name MATLAB stands for matrix laboratory. MATLAB was originally written to provide easy access to matrix software developed by the LINPACK and EISPACK projects. Today, MATLAB engines incorporate the LAPACK and BLAS libraries, embedding the state of the art in software for matrix computation. MATLAB has evolved over a period of years with input from many users. In the university environments, it is the standard instructional tool for introductory and advanced courses in mathematics,

engineering, and science. In industry, MATLAB is the tool of choice for high-productivity research, development, and analysis (Brian, 2001). For this research, we are limited to the Time-Frequency Toolbox.

The Time-Frequency Toolbox is a collection of M-files developed for the analysis of non-stationary signals using time-frequency distributions. This toolbox includes two groups of files: the signal generation files, which allow the synthesis of numerous kinds of non-stationary signals and the processing files, including the time-frequency distributions and other related processing functions (Brian, 2001).

3.2 Methods

The first method in this research work is the generation of signal, in generating the signal of interest; signal with constant amplitude, and with a linear frequency modulation varying from 0 to 0.5 in normalized frequency (ratio of the frequency in Hertz to the sampling frequency, with respect to the Shannon sampling theorem) was considered. This signal is called a chirp, and as its frequency content is varying with time, it is a non-stationary signal. To obtain such a signal, an M-file in an environment of MATLAB was used to generate a linear frequency modulation signal. Consequently, Figure 3.1 highlighted further steps that were followed in achieving the research aim. This was follow strictly followed along the objectives.

A group of methods based on high-order polynomial phase signals are developed to analyze linear frequency modulations, which present higher-order polynomial phase. These methods increase time-frequency resolution and tend to suppress the cross term effect. Some of them are derived from WVD as Polynomial WVD (PWVD), L Class

WVD (LWVD) and based on the Polynomial and L Class WVD (LPWVD) (Wang & Jiang, 2009).

All TFD can be obtained from:

$$TFD(t,w) = \iiint_{-\infty}^{\infty} x\left(u+\frac{\tau}{2}\right) x^{*}\left(u+\frac{\tau}{2}\right) \psi(\theta,\tau) e^{-j\theta t - jw\tau + j\theta u} d\tau d\theta du \qquad (3.1)$$

Where $\psi(\theta,\tau)$is the kernel of the distribution and $x(t)$is the interest signal. When $\psi(\theta,\tau)$=1 the WVD is obtained.

An Electronic Support measures (ESM) system can exploit this information by using two passive receivers, separated in space. Knowing the received waveform will have a sharply peaked ambiguity function, we look at the correlation between the outputs of one receiver, with the output of the second. The correlation will give us the time–delay. We have run simulations over various signal–to–noise ratios (SNRs) with chirp signals. As we are interested in the ability to estimate time–delay at low SNR, we run our simulations with SNR at 12 dB and less.

Form Figure 3.1 the method clearly shown how the research objectives were accomplished. Firstly the signal of interest was selected after which it was generated with complexity of challenges, but the challenges was overcome through appropriate parameter selection which was carried via MATLAB software, thereafter the representation of the signal was carried out Wigner-Ville distribution. Then the Monte Carlo simulation was perfume at various signals to noise ratio and it was compared with Cramer-Rao lower bound (CRLB).

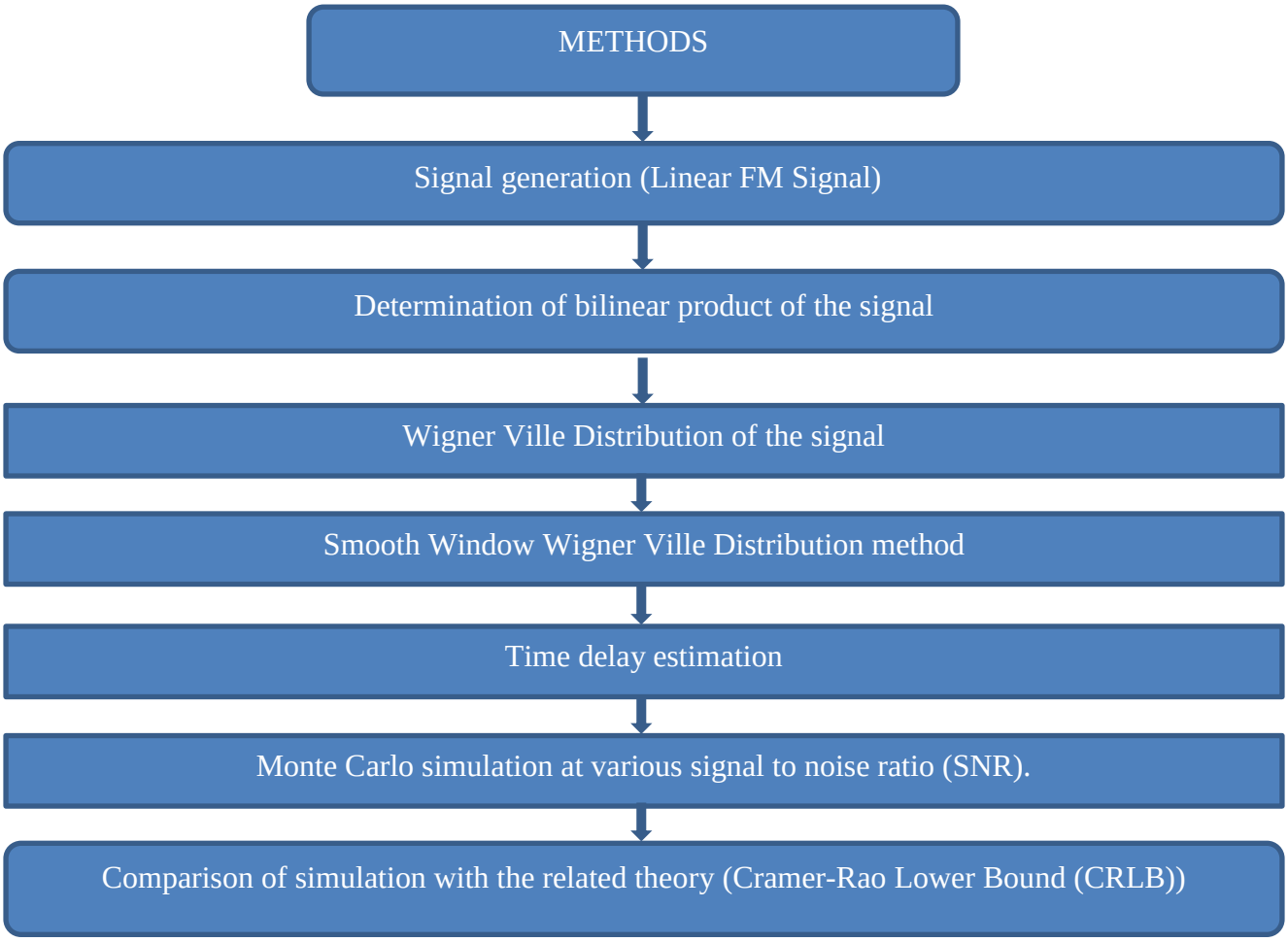


Figure 3.1: Algorithm of the research

3.2.1 Quadratic Time-Frequency Distribution

The QTFD produce energy distribution representation jointly over the time-frequency plane. It also considered as a related class of filtered WVDs with a specific time–lag kernel function (Chee, 2013). The QTFD when expressed with respect to time-frequency kernel and the WVD result as (Boashash, 2016).

$$\rho_z(t,f) = \gamma(t,f) **_{t\ f} W_z(t,f) \tag{3.2}$$

where $\gamma(t,f)$ is a time-frequency kernel and W_z is the WVD. The WVD can be defined as

$$W_z(t,f) = \int_{-\infty}^{\infty} K_z(t,\tau) e^{-j2\pi f\tau} d\tau \tag{3.3}$$

where $K_z(t,\tau)$ is the bilinear product or instantaneous correlation function (IAF).

Cross-terms can cause the interpretation of the TFR difficult. The kernel function is used to overcome this problem. Various kernel functions give rise to a range of different TFDs. Examples of time-frequency kernel are adaptive directional TFD (ADTFD) and adaptive fractional spectrogram (AFS) (Khan & Boashash, 2013; Khan & Boashash, 2015). Some examples of distribution that uses time-lag kernel are Zhao-Atlas-Marks (ZAM) distribution, B-distribution and adaptive smoothed windowed cross WVD (ASW-WVD). For Doppler-lag kernel there are few examples such as modified B-distribution (MBD), adaptive optimal kernel TFD (AOK-TFD) and empirical mode decomposition (EMD) (Kamarulafizam *et al.*, 2013; Stevenson *et al.,* 2010).

The formulation of the QTFD with time-lag kernel is given as

$$\rho_z(t,f)=\int_{-\infty}^{\infty} G(t,\tau)*_t K_z(t,\tau)e^{-j2\pi f\tau}d\tau \tag{3.4}$$

where $G(t,\tau)$ is the time-lag kernel function and the bilinear product, $K_z(t,\tau)$ can be written as (Sandsten, 2013).

$$K_z(t,\tau)=z\left(t+\frac{\tau}{2}\right)z^*\left(t-\frac{\tau}{2}\right) \tag{3.5}$$

where $z(t)$ is the analytical form of the signal. Eq. (3.3) defines the procedure in obtaining smoothed IAF by convolving the time-lag kernel in time with the IAF.

A separable kernel offers independent control of time-smoothing and frequency-smoothing of TFD. Doppler-independent discrete TFD (DI-DTFD) windowed the TFD in frequency direction cause the reduction in inner artifacts while preserving time-marginal. On the other hand, lag-independent discrete TFD (LI-DTFD) smoothed the TFD in time direction causing the suppression of cross-terms while maintaining the frequency marginal. A separable kernel in time-lag function can be defined as (Boashash, 2016).

$$g(t,\tau)=g_1(t)g_2(\tau) \tag{3.6}$$

where $g_1(t)$ is the smoothing function in time and $g_2(\tau)$ is the smoothing function in lag. By using separable kernel, the QTFD in Eq. (3.1) can be described as

$$\rho_z(t,f)=\int_{-\infty}^{\infty} g_1(t)*_{(t)} K_z(t,\tau)\,g_2(\tau)e^{-j2\pi f\tau}d\tau \tag{3.7}$$

Applying a window at $|\tau|\geq T_p$ is sufficient to remove the cross-terms in LFM signal because the auto-terms of the signals are located between $0\geq|\tau|\geq T_p$ while the cross-terms

are positioned between $T \geq |\tau| \geq T + T_p$ (recall that $T >> T_p$). The position of the auto-terms and cross-terms are same with the LFM signal (Sandsten, 2013).

Correspondingly, Equation (3.6) when expressed in the ambiguity domain is as follows

$$A_z(v,\tau) = \underset{\substack{t \to v \\ f \to \tau}}{FT}\left[\rho_z(t,f)\right] = \underset{\substack{t \to v \\ f \to \tau}}{FT}\left[g(t,f) \underset{(t)\,(f)}{**} W_z(t,f)\right] = G_1(v) A_z(v,\tau) g_2(\tau) \qquad (3.8)$$

3.2.2 Cross terms

The Wigner distribution gives cross-term that is located in the middle between the auto terms. And it does not matter how far apart the different signal components are, cross-terms show up anyway and between all components of the signal as well as disturbances in the case of noise. This makes the interpretation of the Wigner Ville distribution difficult and to analyse the signal it was important to eliminate the cross-terms which was carried out appropriately with help of appropriate MATLAB function.

As the WVD is a bilinear function of the signal x, the quadratic superposition principle applies. Unlike the spectrogram interference terms, the WVD interference terms was a non-zero regardless of the time-frequency distance between the two signal terms. These interference terms were troublesome on trying to analyse the signal as a result of the overlap with auto-terms (signal terms) and thus make it difficult to visually interpret the WVD image. However, it appears that these terms must be present or the good properties of the WVD (marginal properties, instantaneous frequency and group delay, localization, unitarily) cannot be satisfied. Actually, there is a trade-off between the quantities of interferences and the number of good properties.

The prominent feature of the Wigner Ville distribution is high resolution in both time and frequency due to the absence of a window in calculating the time frequency representation (Ralston *et al*, 2007).

3.2.3 Pseudo and Smoothed-Pseudo Wigner-Ville Distribution

One of the simple methods is the Pseudo WVD. Here, a filtering window is introduced, whereby the result is filtered in the frequency plane, and the interferences formed by distantly placed components in the time plane are suppressed. However, cross terms created by distant auto-terms in the frequency plane cannot be removed by frequency filtering. For the removal of the cross terms formed in the frequency plane, windowing in time is needed. In consequence of this fact, smoothing in time and frequency was introduced. SPWVD enables the reduction of all types of interferences at the cost of resolution reduction. Smoothing by wider time filtering (window width Lg) and frequency filtering (window width Lh) leads to interference reduction and slow degradation of the resolution (Abdullah *et al.*, 2011).

3.2.4 Signal Characteristics in Ambiguity Domain

There are many ways in implementing cross-terms suppression techniques as described in the literature. Some of the earlier works applied the cross-terms suppression in ambiguity domain (Doppler-lag domain) in the effort of producing accurate TFR. For radar signals, the cross-terms in the time-frequency domain are highly oscillatory and are located away from the origin in the ambiguity domain while the auto-terms are located close to the origin. Hence, the kernel function that focuses on preserving the energy located close to the origin should be able to remove most of the cross-terms.

Radially Gaussian kernel (RGK) is one of the earliest techniques that incorporates the removal of cross-terms in the ambiguity function. Later this kernel function is improved as adaptive optimal kernel TFD (AOK-TFD) that is able to produce accurate signal representation even in low SNR. The AOK-TFD is limited to linear FM signal for an accurate TFR and is significantly computational expensive. Many other works have considered the implementation of kernel function in the ambiguity domain such as cone shaped kernel (CSK), high energy concentration (HEC) kernel, and radial Butterworth kernel (RBK).

3.2.5 General Signal Characteristics in IAF

Assuming the received signal is expressed as

$$\begin{aligned} x(t) &= x_1(t) + x_2(t) \\ &= x_1(t) + x_1(t-T) \end{aligned} \tag{3.9}$$

where usually the analytic signal $z(t)$ is used, without any restrictions $z(t)$ could be replaced by $x(t)$.

The Instantaneous autocorrelation function (IAF) defined in Equation (3.5) can be viewed as the correlation of signal itself over successive time intervals. Substituting Equation (3.9) into Equation (3.4) produces the following equation

$$K_z(t,\tau) = \left[z_1\left(t+\frac{\tau}{2}\right) + z_2\left(t+\frac{\tau}{2}-T\right)\right]\left[z_1^*\left(t-\frac{\tau}{2}\right) + z_2^*\left(t-\frac{\tau}{2}-T\right)\right]$$
$$= \underbrace{K_{z,11}(t,\tau) + K_{z,22}(t-T,\tau)}_{auto-terms} + \underbrace{K_{z,12}\left(t-\frac{T}{2},\tau+T\right) + K_{z,21}\left(t-\frac{T}{2},\tau-T\right)}_{cross-terms} \quad (3.10)$$

By referring to Equation (3.10), the auto-terms represent the desired signal characteristics that lie along the time axis (at τ=0, first and second term) while the cross-terms located at a lag of $|\tau|>0$ (third and fourth term) introduce interference in the TFR. Thus, it is important to preserve the auto-terms and attenuate the cross-terms.

Cross-terms in Equation (3.10) are produced by the correlation between first signal, z_1 and second signal, z_2. The cross-terms always lie midway between corresponding signal components (Hlawatsch, 1984). By referring to Equation (3.10), a kernel must have the ability to filter out terms at $|\tau|>0$. This property is crucial in designing a cross-terms free representation.

3.2.6 Analysis of Signal in the Presence of Noise

This section deal with some of the analysis tools introduced to aid the accuracy of parameter estimation in the presence of noise. The noise is modeled by a common form of noise in signal processing, Additive White Gaussian Noise (AWGN) that has a normal probability density function of approximately zero mean and unity variance.

3.2.7 Selection of Appropriate Window Function

There are many varieties of windows function, but only one was considered in this research work to test the effect of correct estimation; hamming window was adjustable parameter, the window length N.

3.2.8 Instantaneous frequency (IF) Estimation

In many practical applications, the received signal is time-varying which means the signal frequency is time-dependent. Instantaneous frequency (IF) estimation can be used to describe the frequency characteristics of the signal. Normally, a good estimator is the one that is consistent and is statistically and computationally efficient. There are many IF estimate algorithms presented in the literature which can be grouped into phase modeling methods, time-frequency methods, signal modeling methods and phase differencing methods (Boashash, 2003).

Rao and Taylor proved that WVD peak based IF estimation is optimal for linear FM signals for moderate and high SNR although the estimator performance degrades significantly at low SNR. Peak based IF estimator can be expressed as

$$f_i(t) = \arg\left[\max_f \rho(t, f)\right] \tag{3.11}$$

where $f_i(t)$ is the IF and $\rho(t,f)$ is the TFD. Peak based IF estimator is able to produce a decent estimation as its capability to localize energy along the IF law. In this thesis, peak TFD technique is used for IF estimation as it is easy to implement.

3.2.9 Cramer-Rao Lower Bound

For the purpose of measuring the performance of any unbiased parameter estimator, the Cramer–Rao Lower Bound (CRLB) is frequently used because it can provide the theoretical limit to the variance of the estimator. The most efficient estimator is the one

that can achieve the lower bound on the variance. The general formulation for CRLB for IF estimate is

$$\mathrm{var}\left(\hat{f}_l\right) \geq \frac{24}{(2\pi)^2 \gamma N\left(N^2-1\right)} \tag{3.12}$$

where N is the average window width, and γ is the SNR. With the assumption that the real IF of the signal is known, the variance of the IF estimate from measurement can be expressed as

$$\mathrm{var}\left(\hat{f}\right) = \frac{1}{N}\sum_{n=0}^{N-1}\left(\hat{f}_n - f\right)^2 \tag{3.13}$$

where N is the total number of samples, f is the real IF and $\hat{f}_n$ is the estimated IF at every n.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Results and Discussion

4.1.1 Time and Frequency representation

Figure 4.1 shows the time-domain representation $x(t)$ as earlier mention in literature review, with constant amplitude and with a linear frequency modulation in a normalized frequency with the ratio of the frequency in Hertz to the sampling frequency, taking into consideration Shannon sampling theorem. This kind of signal is what we call chirp signal, which is the signal of interest in this research work and its frequency content is varying with time, it is also a non-stationary signal, this was obtained with the help of MATLAB function as indicated in appendix A. From this time-domain representation, it is difficult (except in some case for experienced specialists) to say what kind of modulation is contained in this signal, what are the initial and final frequencies, is it a linear, parabolic, hyperbolic frequency modulation and so on and so forth (François, 1996).

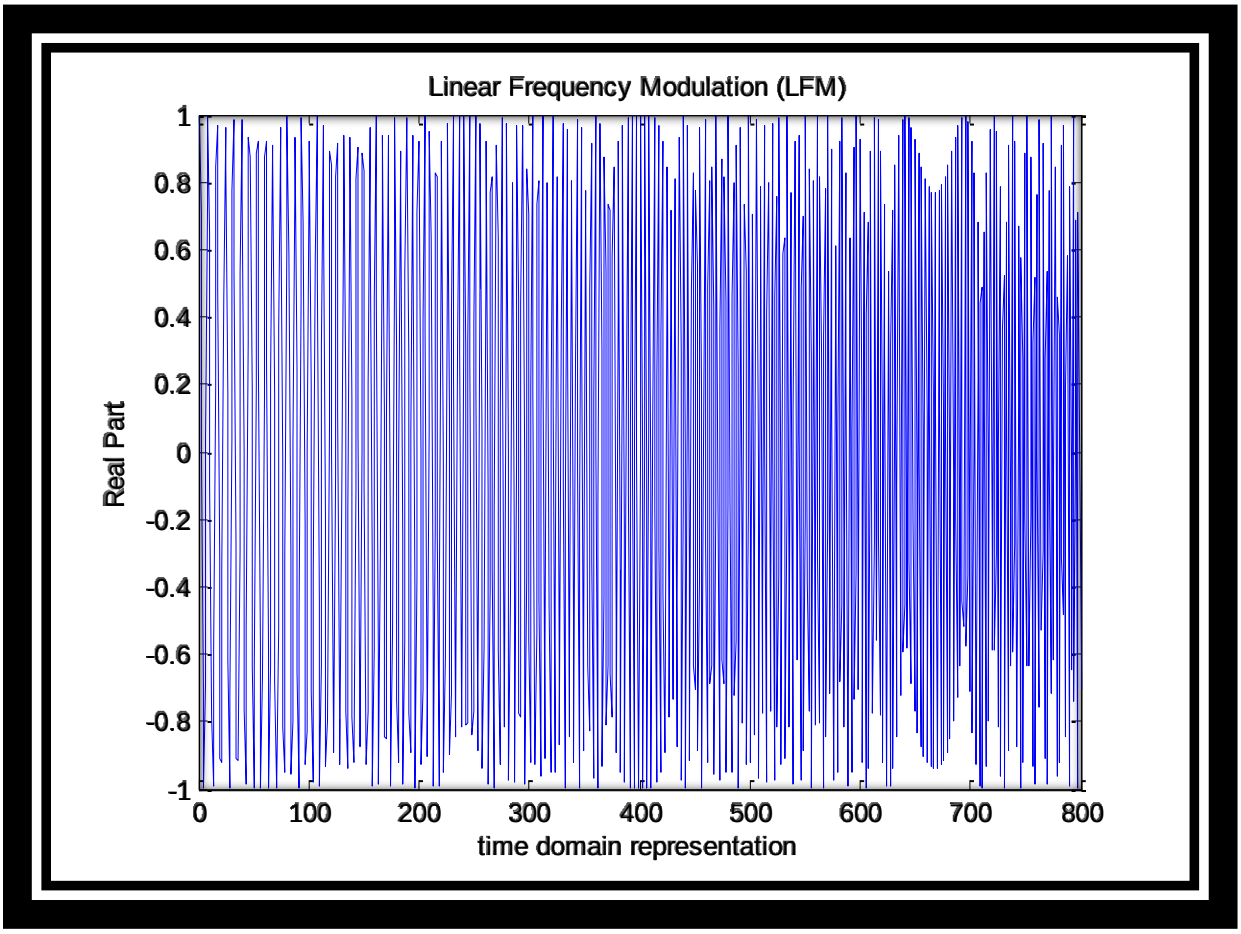


Figure 4.1: Time domain representation of Linear Frequency Modulation (LFM) (Chirp signal)

Considering the energy spectrum of this signal by squaring the modulus of its Fourier transform (using the fft function) in the MATLAB, as shown in Figure 4.2, we still cannot say, from this plot, anything about the evolution in time of the frequency content. This is due to the fact that the Fourier transform is a decomposition on complex exponentials, which are of infinite duration and completely un-localized in time domain. From figure 4.2, it is clear that the time information is in fact encoded in the phase of the Fourier transform (which is simply ignored by the energy spectrum), but their interpretation is not straightforward and their direct extraction is faced with a number of difficulties such as phase unwrapping (François, 1996).

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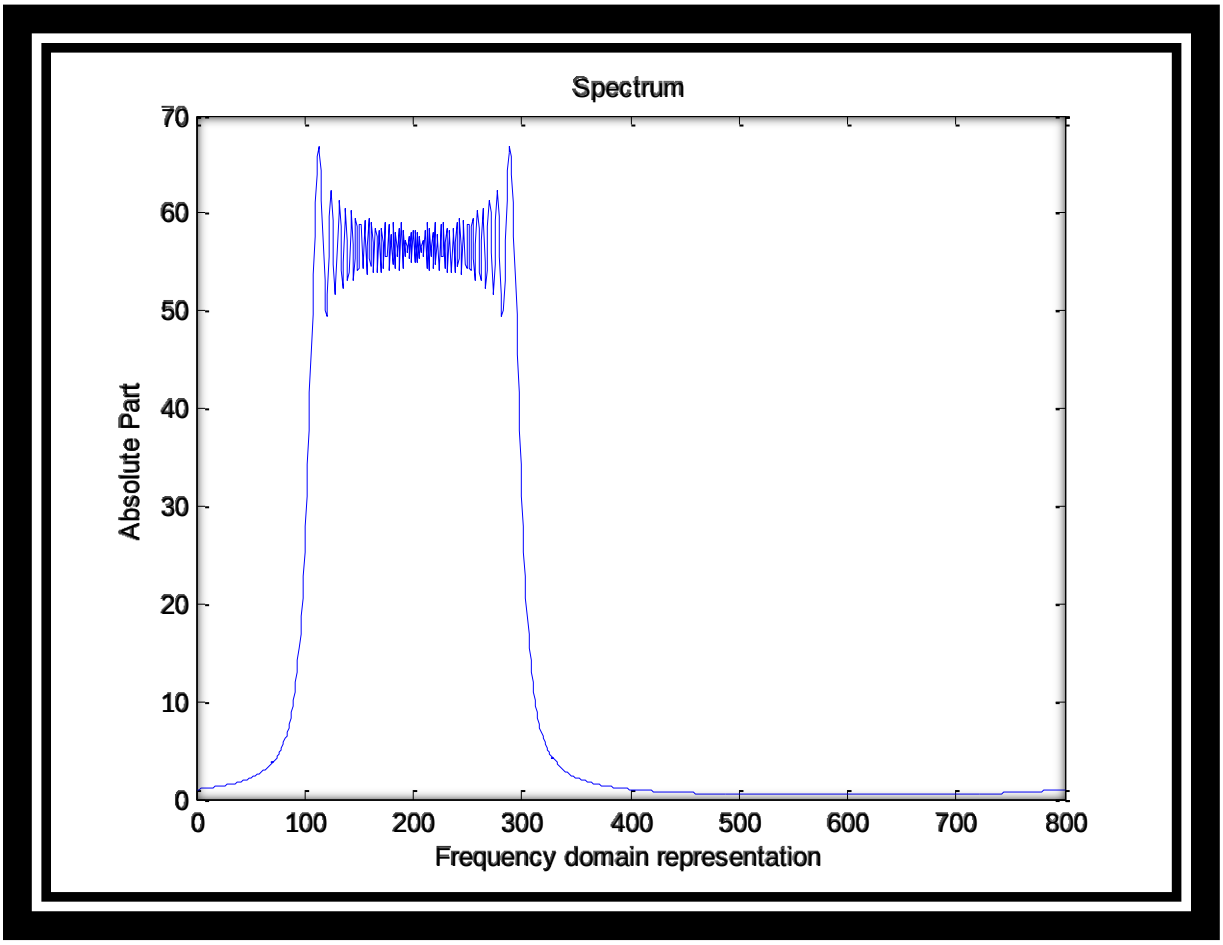


Figure 4.2: Frequency domain representation of Linear Frequency Modulation (LFM) (Chirp)

Figure 4.3 is delayed version of figure 4.1 that is to say the signal is delayed for some sometime then it continues.

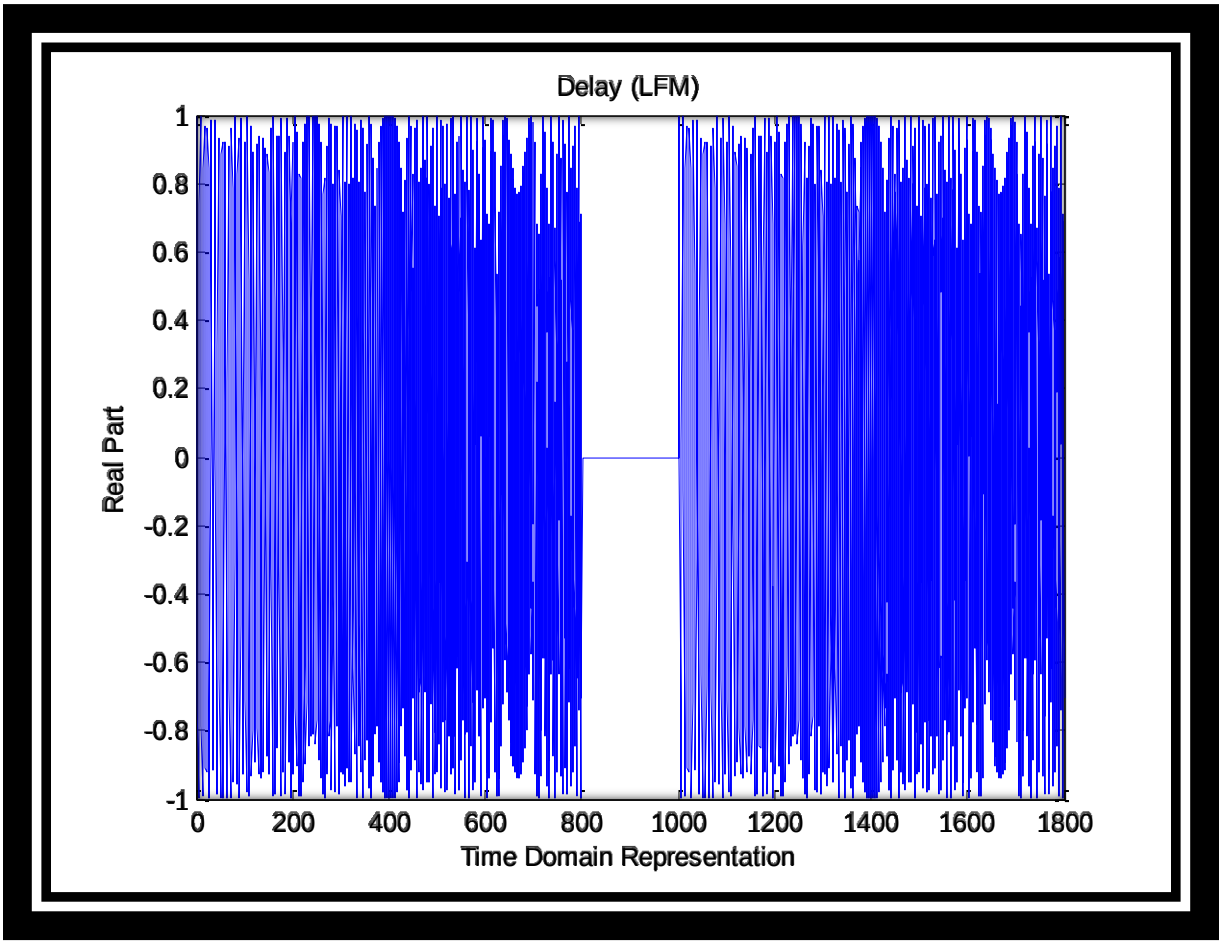


Figure 4.3: Delayed version time domain representation of Linear Frequency Modulation (LFM) (Chirp)

Figure 4.4 is the energy spectrum of this signal in Figure 4.3 which was obtain by by squaring the modulus of its Fourier transform (using the fft function) in the MATLAB, as shown in Figure 4.4, we still cannot say in any way from the plot, anything about the evolution in time of the frequency content. This is due to the fact that the Fourier transform is a decomposition on complex exponentials, which are of infinite duration and completely un-localized in time domain. From Figure 4.4, it is still clear that the time information is in fact encoded in the phase of the Fourier transform (which is simply ignored by the energy spectrum), but their interpretation is not straightforward and their direct extraction is faced with a number of difficulties such as phase unwrapping (François, 1996).

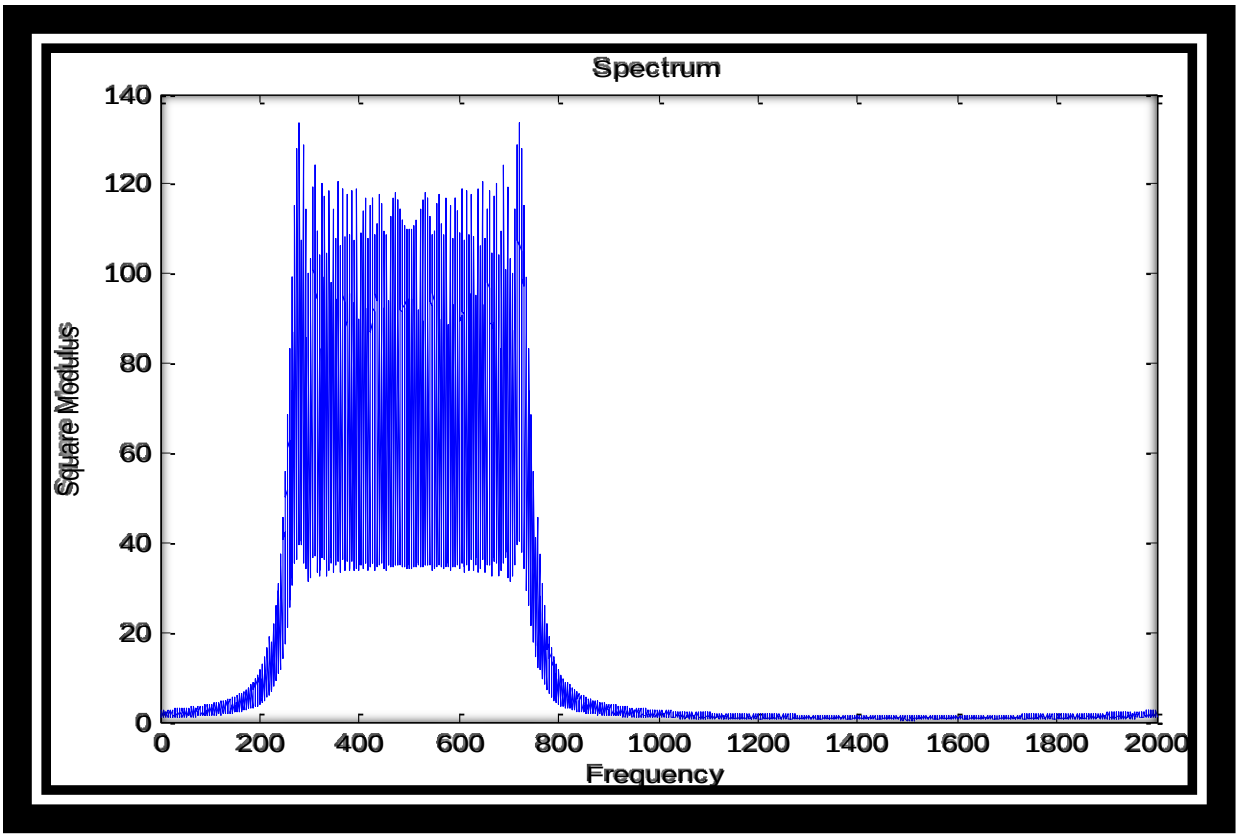


Figure 4.4: Frequency-Domain Representation of the signal

However from Figure 4.1 to Figure 4.4, the generation of signals and their corresponding energy spectrum are carried out in a noise free environment and is achieved by not introducing any form of noise in the generation of the signals and the energy spectrum. For the purpose of comparison, noise was later introduces in the generation of the signal which resulted to Figure 4.5 and the energy spectrum in Figure 4.6 respectively. It is worse than before to interpret these plots.

The time representation is usually the first (and the most natural) description of a signal we consider, since almost all physical signals are obtained by receivers recording variations with time. The frequency representations, obtained by the Fourier transform is also a very powerful way to describe a signal, mainly because the relevance of the concept of frequency is shared by many domains (physics, astronomy, economics, biology) in which periodic events occur.

But if we look more carefully at the spectrum $X(f)$, it can be viewed as the coefficient function obtained by expanding the signal $x(t)$ into the family of infinite waves, which are completely un-localized in time. Thus, the spectrum essentially tells us which frequencies are contained in the signal, as well as their corresponding amplitudes and phases, but does not tell us at which times these frequencies occur.

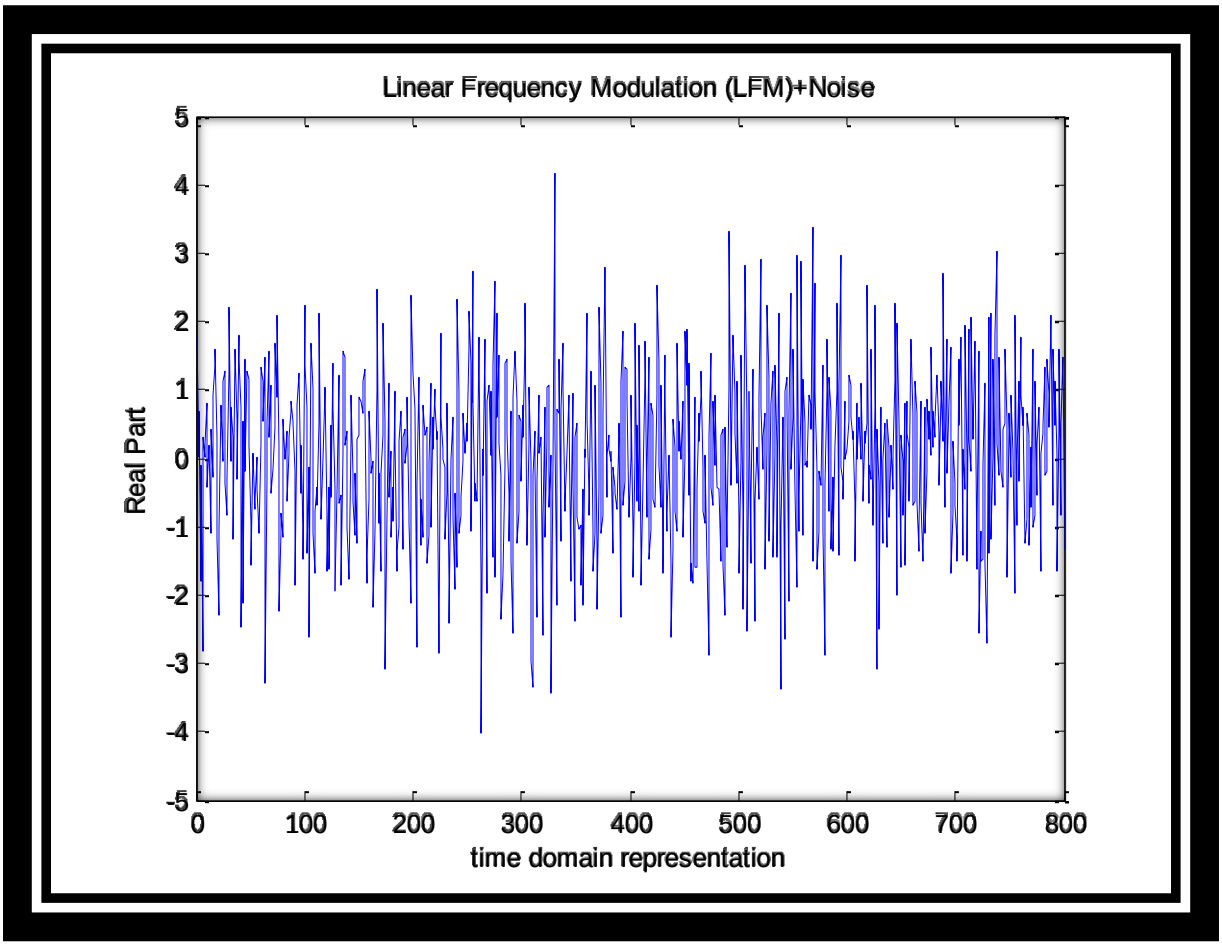


Figure 4.5: Time domain representation of Linear Frequency Modulation (LFM) (Chirp) in the presence noise

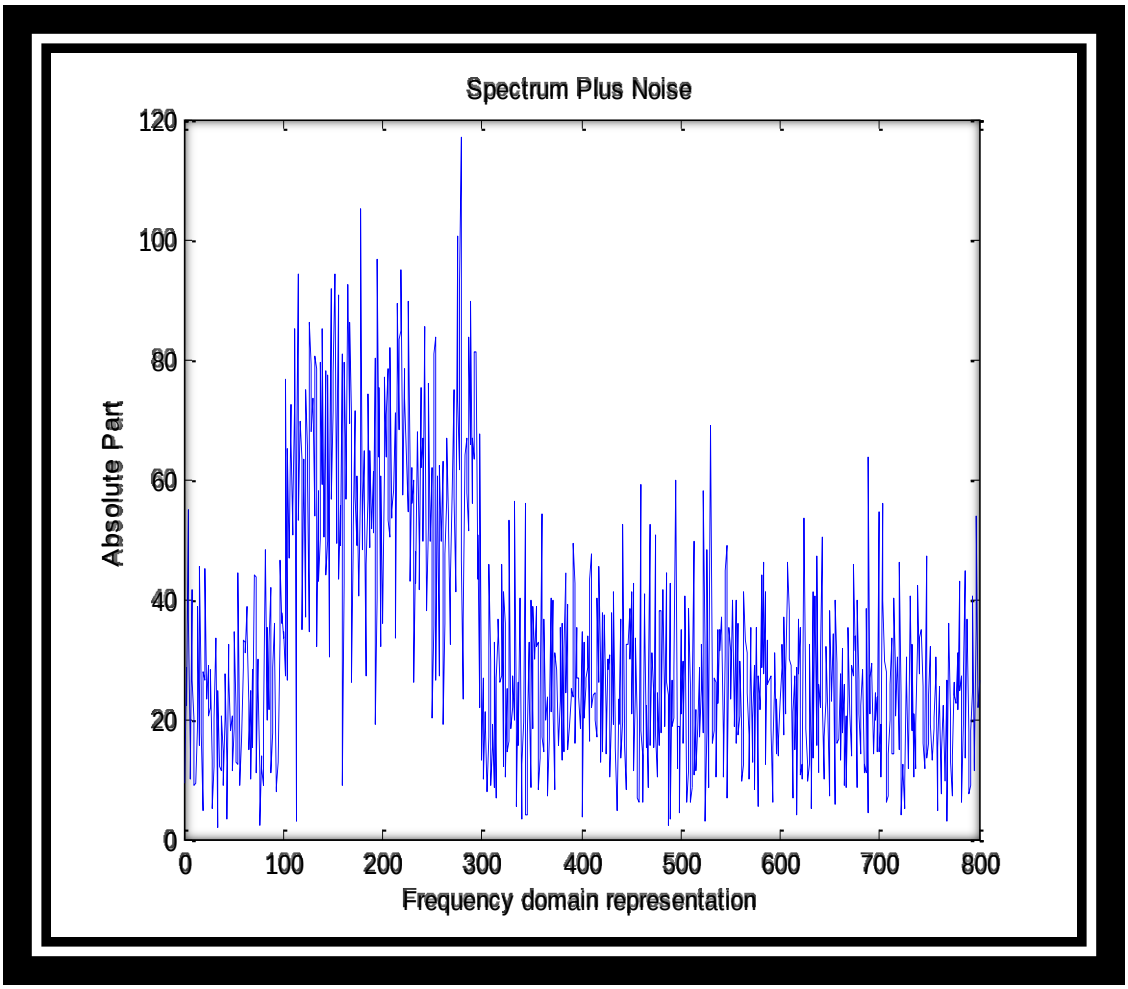


Figure 4.6: Energy spectrum of the noisy chirp

4.1.2 Quadratic Time-Frequency representation

Since the conventional representations in the frequency domain or time domain are insufficient in the circumstances as discussed in review, the obvious solution was to seek an alternative representation of the signals as a two-variable function or distribution whose domain was the two-dimensional(t, f)space. Its constant-f cross-section should show the time or times at which frequency f is present and on the other hand its constant-t cross-section should show the frequency or frequencies present at time t. such kind of representation is what we called a time-frequency representation (TFR) or time frequency distribution (TFD). For effective and efficient informative analysis of the research work, it was important we directly represent the signal's frequency content while still keeping the time description parameters which lead us to the used of time-frequency representations of signals as indicated in Figure 4.7 and 4.8 respectively. However, the figures are considered under noise free environment and it's indicate a cross-term in between which makes it difficult to interpret and visualized the time-frequency plane. For Figure 4.7, it appears in the center the cross term with a high degree of spread while in Figure 4.8 there is reduction in the spread of the cross-terms which pave way for less improvement in the analysis of the signal even though not with a higher degree but at least there is improvement and is as result of parameter selection.

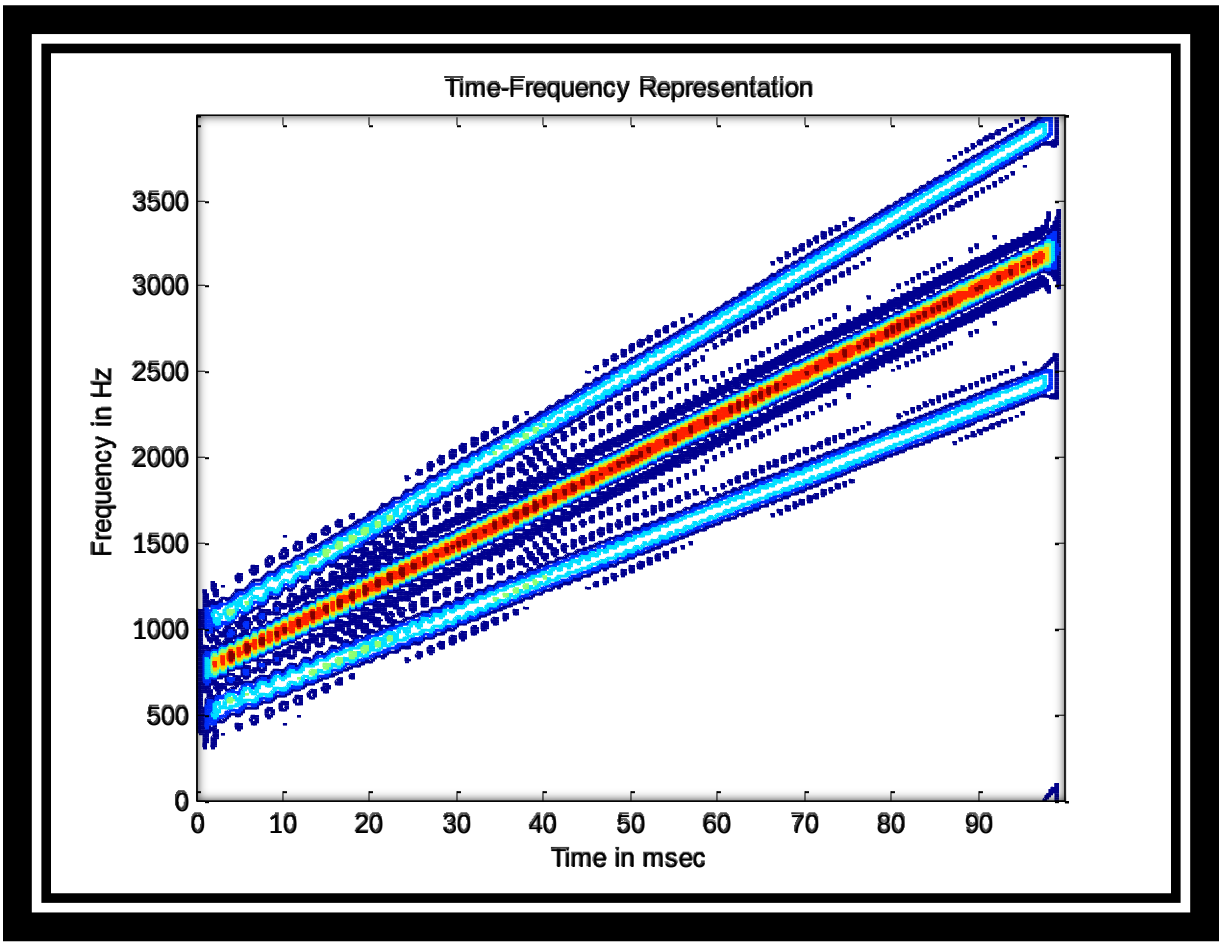


Figure 4.7: Time Frequency representation of the signal with Cross Term

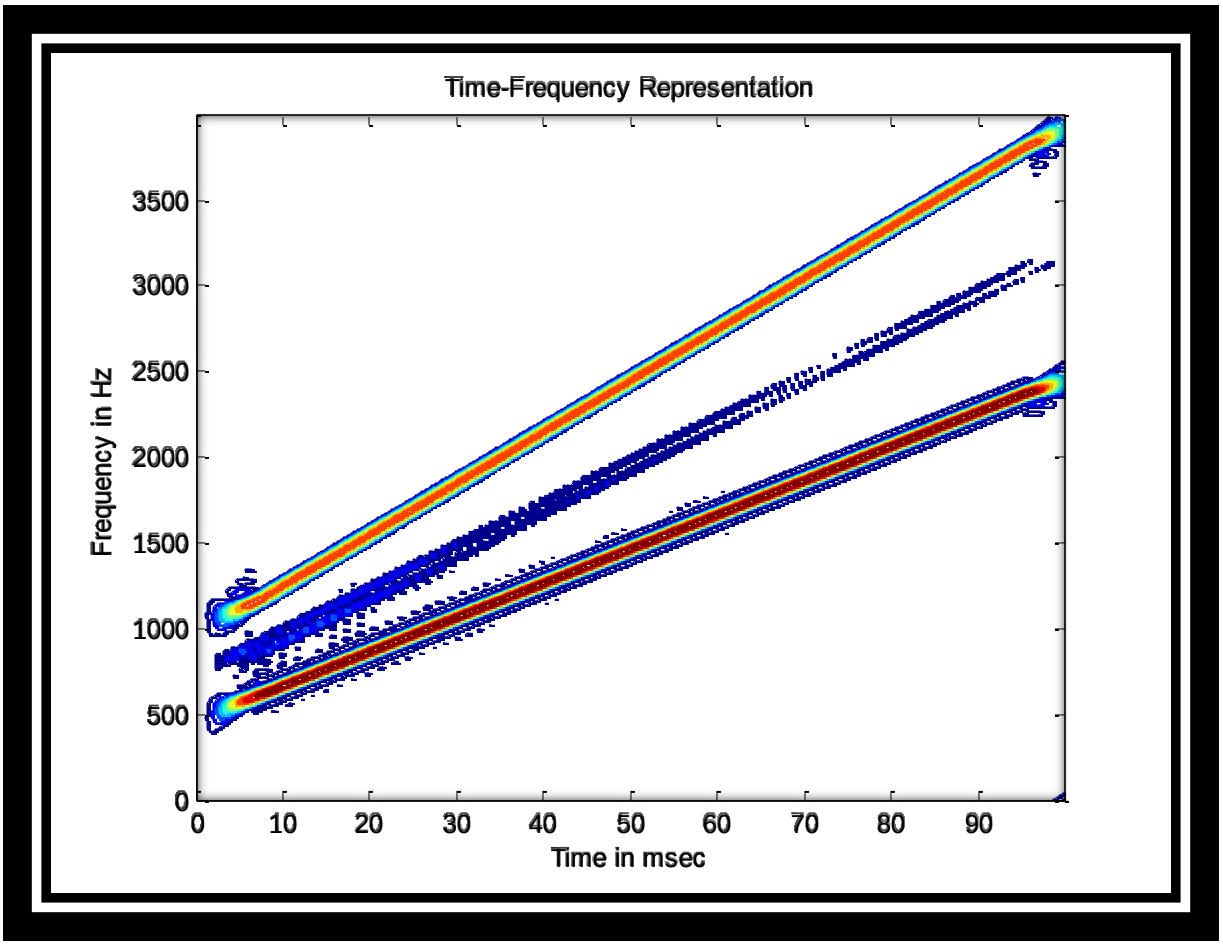


Figure 4.8: Time Frequency representation of the signal with Cross Term

They provide information which ordinarily are not available in frequency or in time alone, the time-frequency representation helps to provide information about auto-components of the signals, it define their time duration and frequency bands over the components which indicates the signals by the sides and the cross-term at the middle (Hlawatch & Boudreaux-Bartels, 1992 and Shafi *el al*., 2009). More so, introducing noise in to the system makes it worse for analysis and interpretation as shown Figure 4.9 with a high degree of noise and Figure 4.10 with less degree of noise compare to Figure 4.9 respectively.

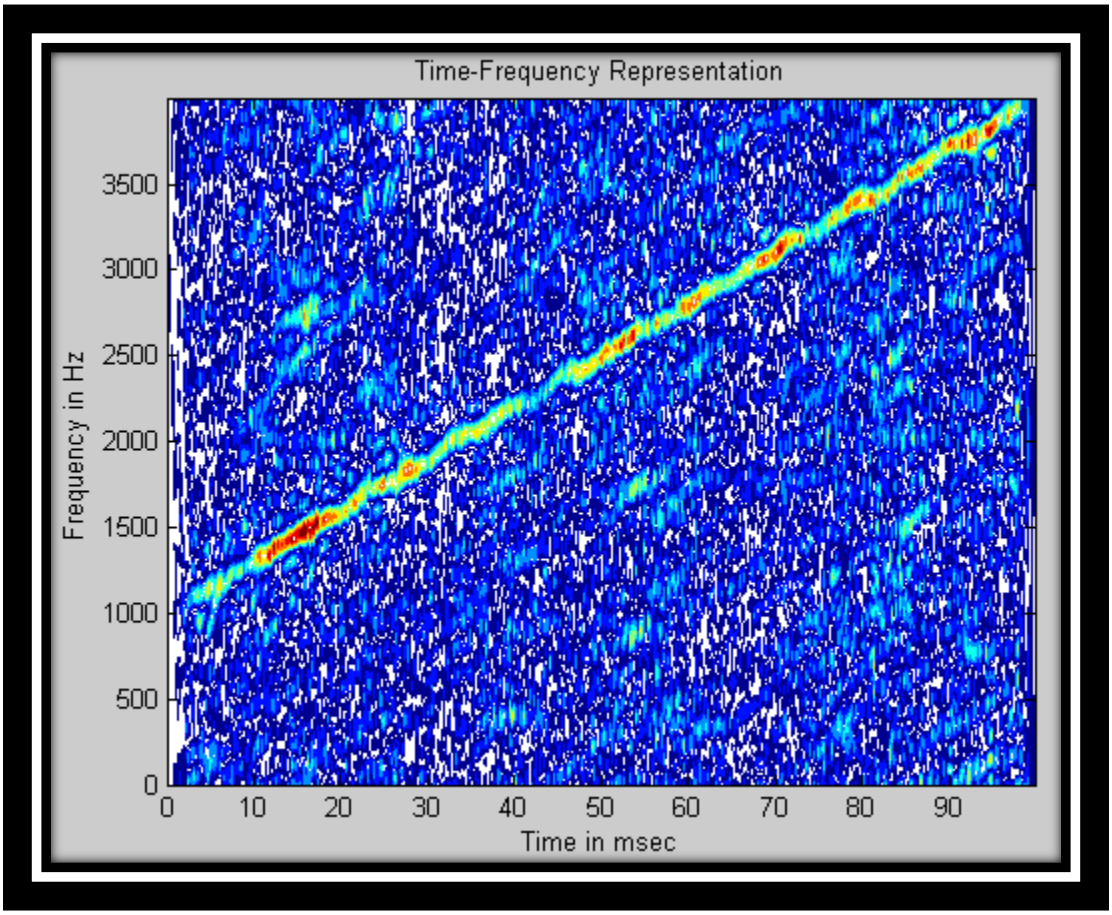


Figure 4.9: Time Frequency representation of the signal with Cross Term in presence of noise

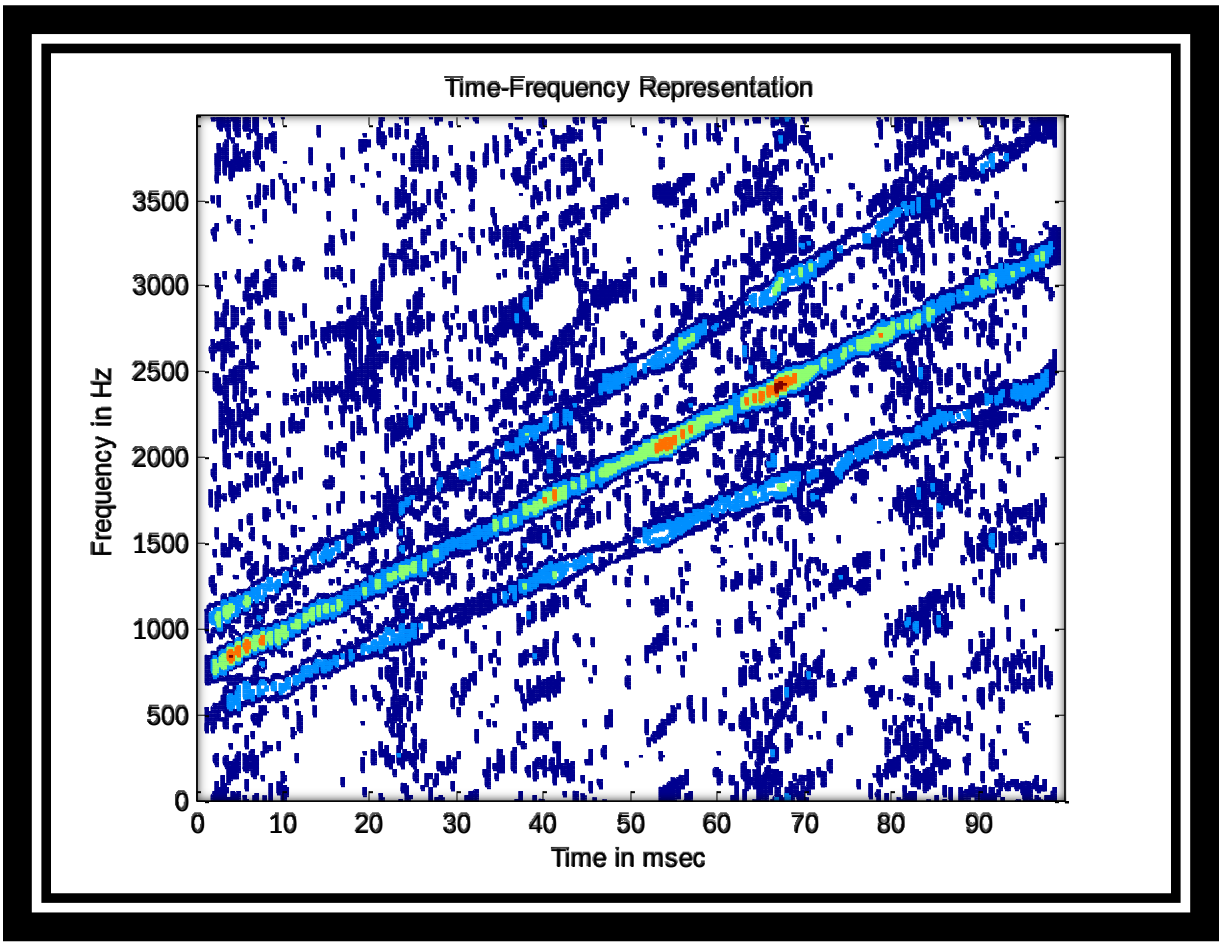


Figure 4.10: Time Frequency representation of the signal with Cross-term in presence noise

4.1.3 The Wigner-Ville Distribution

It is clear from the review and proven from the simulation results as shows Figure 4.11 and 4.12 respectively that quadratic time-frequency (Wigner-Ville Distribution) representation offers better resolution of the auto-components even though it has a challenge of cross-term which appears at center of the signals. However, choice of time-frequency representation depends on the area of application.

The result of the Wigner Ville distribution shows that a quadratic TFD is obtained by smoothing the symmetric ambiguity function (using the kernel function) and then by taking a two-dimensional (2-D) Fourier transform of the result as obtained by the MATLAB function indicated in Figure 4.11, and 4.1 respectively. This operation is equivalent to a 2-D filtering in the ambiguity domain. Thus, by a proper parameter selection, we were able reduce or remove the cross terms. The filtering of the cross terms in the ambiguity domain that reduces (or removes) the cross terms in the time–frequency domain unfortunately results in a lower time–frequency resolution. That is, there is tradeoff between cross term suppression and time–frequency resolution in the design of a given quadratic TFD.

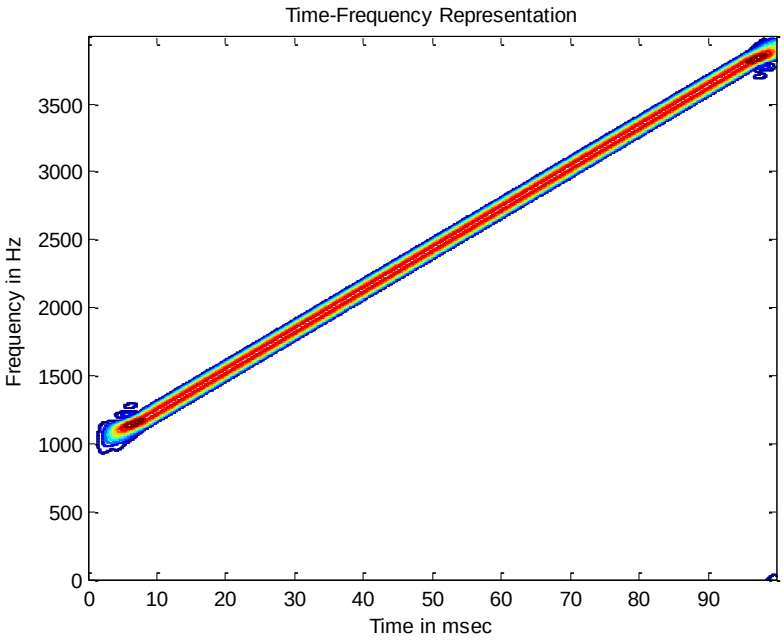


Figure 4.11: Wigner-Ville distribution of the chirp

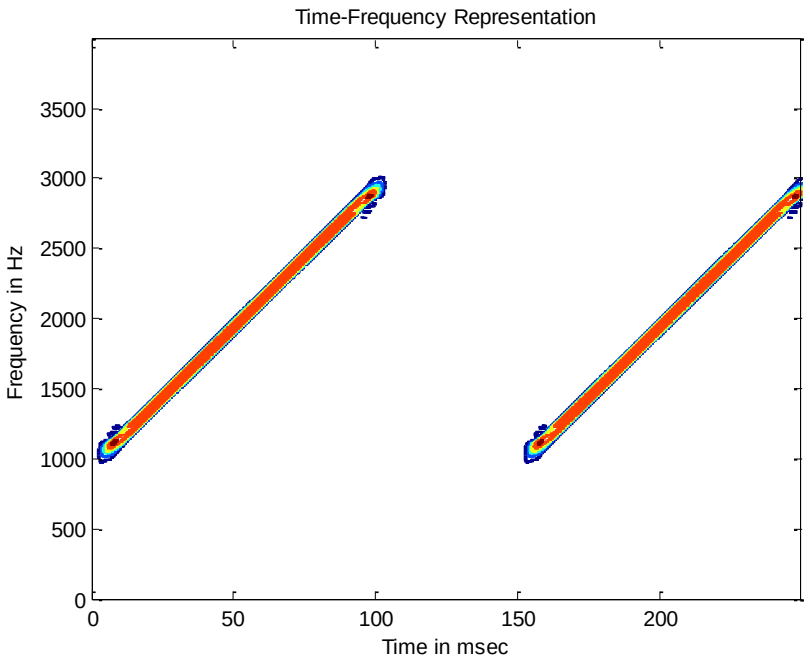


Figure 4.12: Delayed Wigner-Ville distribution of the chirp

4.1.4 Monte Carlo Simulation

A sort of Monte Carlo simulation was carried out to compare the simulation result with that of the theory. For validation of the method, the Monte Carlo simulation results were presented graphically as shown in Figure 4.13 to 4.15 and Figure 4.16 and 4.17 respectively. In carrying out the simulation and comparing with the theory, the results are presented in two different forms. The first form of the result agrees with the theory while the other form did not agree. The value of the SNR varies from 0dB to 12dB with an interval 2dB. At each level of the SNR, 100 to 1000 runs of Monte Carlo simulation were carried out, and their results were shown in Figure. 4.13 to 4.15 and Figure 4.16 and 4.17 respectively. As shown in Figure 4.13 to 4.15, for the range of the signal components, when the SNR value is within 5 to 6dB, the simulation results and that of the Cramer Rao Lower Bound (CRLB) (theoretical) are matched. But on the way round as shown in Figure 4.16 and 4.17 the simulation result leads that of theory, that is to say there is disagreement between simulation and theory. However, the reason could be that approximations are no longer valid at that point of the simulation.

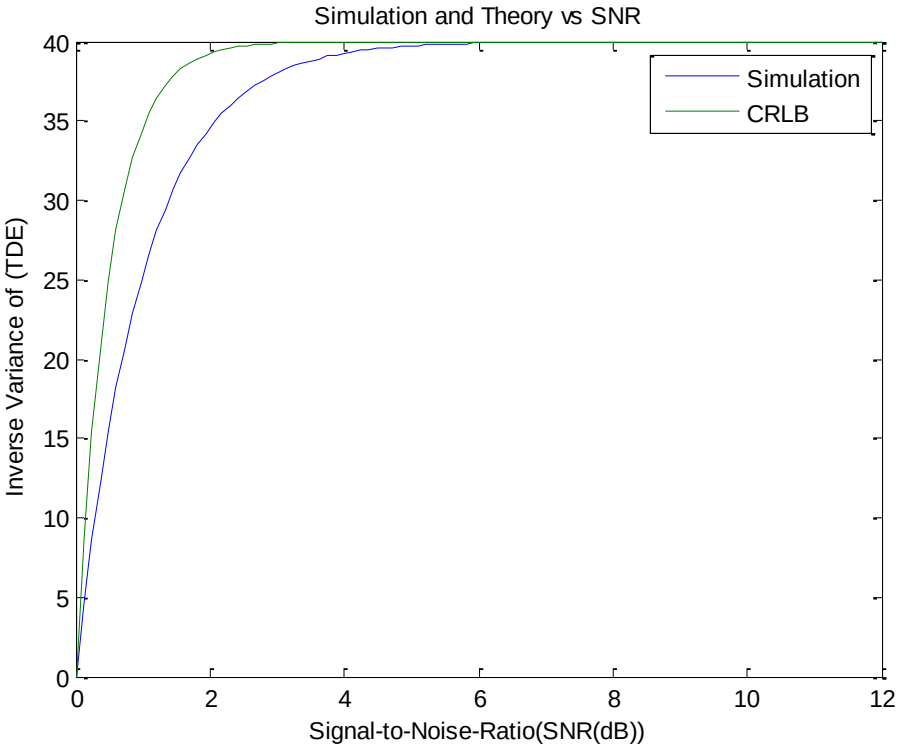


Figure 4.13: Delay Simulations versus CRLB

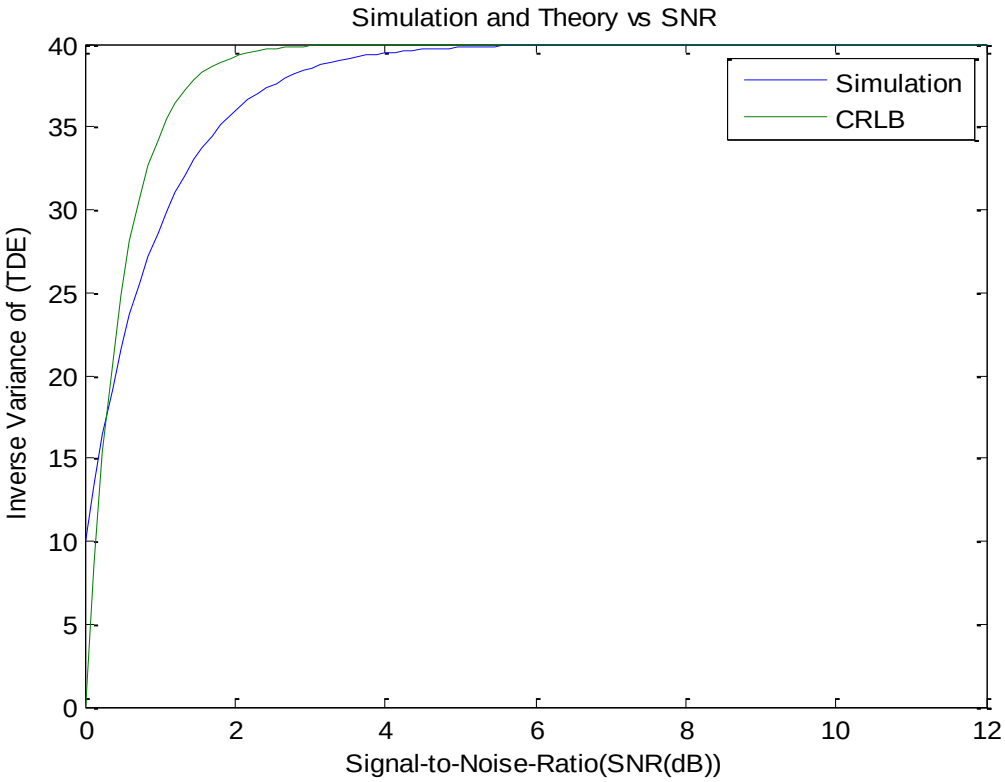


Figure 4.14: Delay Simulations versus CRLB

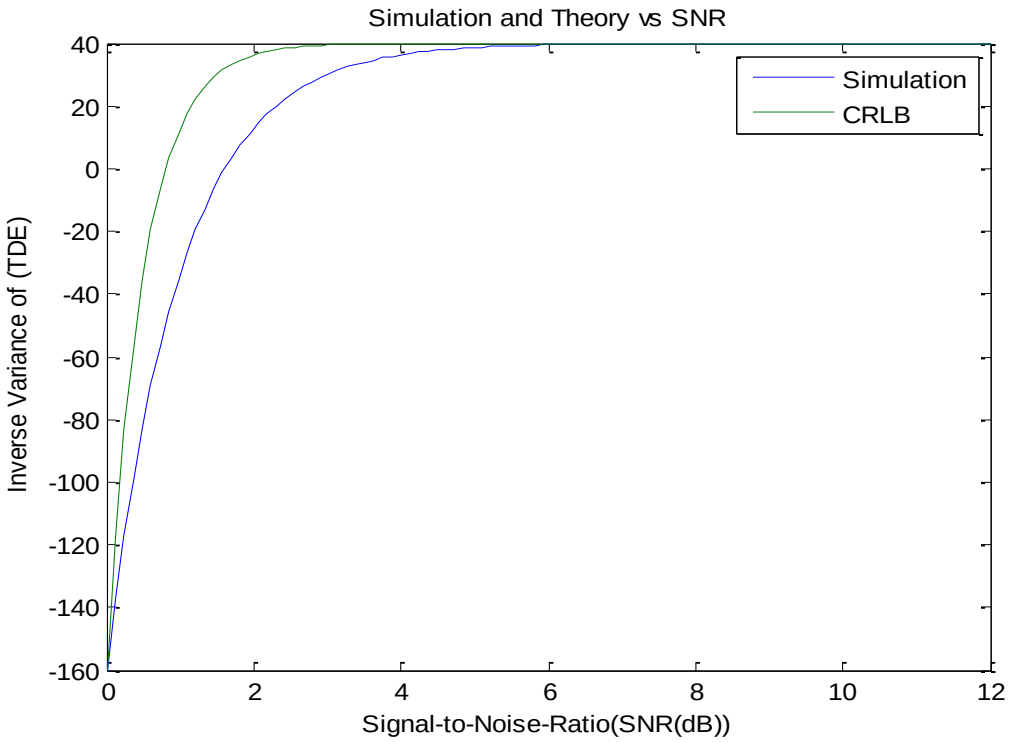


Figure 4.15: Delay Simulations versus CRLB

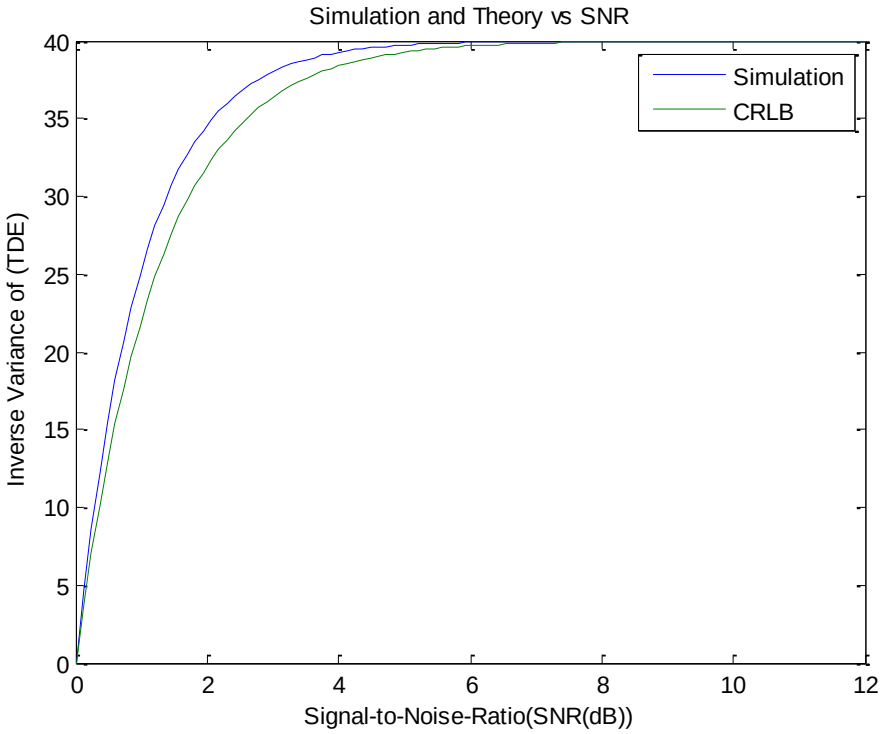


Figure 4.16: Delay Simulations versus CRLB

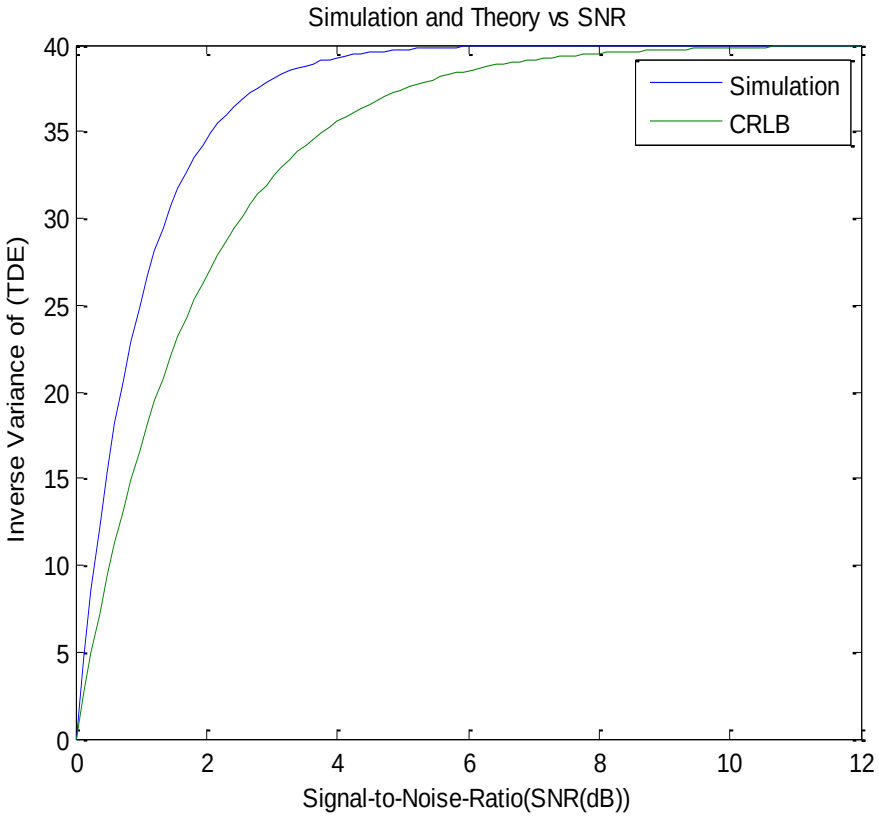


Figure 4.17: Delay Simulations versus CRLB

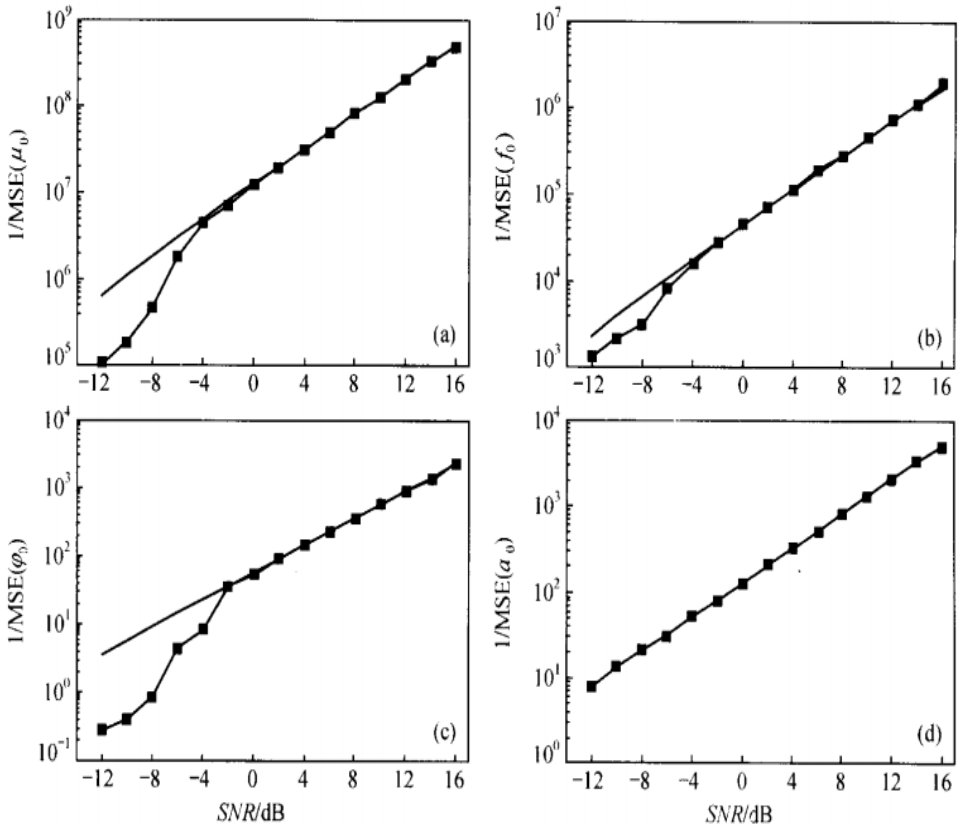


Figure 4.18 MSEs of the parameter estimates of the first component – Theoretical results ■Simulation results

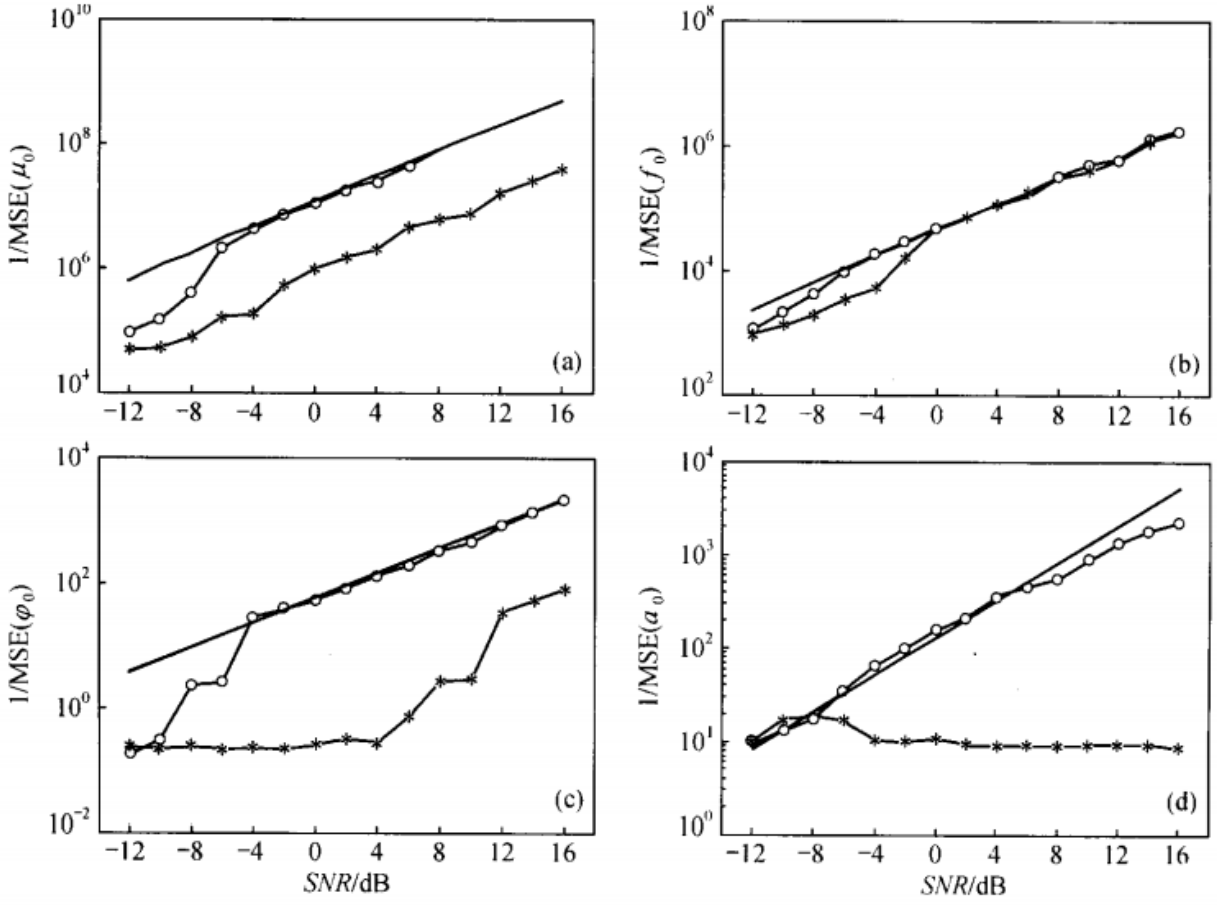


Figure 4.19 MSEs of the parameter estimates of the second component – Theoretical results, results direct detection* results of the detection with signal separation

Qi *et al,* (2004) in their simulations results as shown in Figure 4.18 and 4.19 respectively, for the first signal component, when the input SNR is higher than -2 dB, the simulation results and the theoretical analyses are well matched and the means square errors (MSEs) are not more than 1.2 times of the CRLBs. As the input SNR decreased, there will be disagreements between the simulation results and the theoretical analyses. Obviously, this is because the approximations implied in the theoretical analyses are no longer valid at low SNRs.

For the second signal component, the detection and parameter estimation are disturbed by the stronger component. Figure 4.19 shows that the estimation accuracies of a_0, φ_0 and μ_0 degrade dramatically when no measure is taken to reject this disturbance. Compared with the results shown in Figure 4.18, for the same input SNR, the MSEs of estimates will enlarge about 100 times for a_0, φ_0 and about 10 times for μ_0, but the estimation of $f0$ will be influenced slightly and the MSE will increase about 2 times. By suppressing the first component with the signal separation technique, the estimate accuracies are improved effectively and the MSEs get close to the results shown in Figure 4.18. Analysis of the simulation results indicates that, for the SNR higher than -2 dB, the MSE of each estimate of the second component is not larger than two times the CRLB. The simulation results compared with our simulations results are in agreement to some extent which shows our validation of the results.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

In this research work, time domain and frequency domain representation were viewed and analyze as defined by Fourier transform. The limitation posed by separate representation of signal in time domain and frequency domain and also highlighted the motivation of joint time-frequency description of signals by means of time frequency representation (TFR). Time-frequency signal representations characterize signals over a time-frequency plane. They thus combine time domain and frequency domain analysis to yield a potentially more revealing picture of the temporal localization of a signal's spectral components

The research work also reviews several linear and quadratic TFRs, discuss their motivation and their properties, with an emphases placed on the basic properties, running window and filter interpretation, time resolution versus frequency resolution for better analysis. The research work more so shows the theory of time delay estimation was formulated with the use of the linear frequency modulated (LFM) signal, represented in quadratic form, the quadratic form offers better resolution of auto-components but have cross-terms. The TFD not only shows the start and stop times and the frequency range, but also clearly shows the variation in frequency with time. However, the drawback set out by such representation, necessary techniques were put in place to overcome its short coming.

5.2 Conclusion

The research presented in this thesis has widened the application of the joint time-frequency analysis for the analysis of linear frequency modulation (LFM) signal.

The new proposed TFD kernels provided an improvement on the use of TFD to the previous work as reported in literature which normally focused on a limited class of signal. The QTFD developed in this research wok was capable of performing signal analysis for a wide class of signals including linear FM and the pulse signal. Delay estimation was face with a lot of challenges ranging from preferable signal to be used, form of signal representations and so on and so forth. But all this challenges was overcome with the help of using LFM signal because of its high resolution and it also have the advantage of improving target discrimination compare to simple pulse radar. Wigner Ville distribution as a high-resolution spectral deformation composition method, however its application to delay estimation is limited due to cross-term effects, but the help of appropriate parameter selection from MATLAB it was able to suppress all forms of disturbance and interference which pave way for better analysis and delay estimation.

The simulation shows the time delay compared with the theory (CRLB), which shows at a point the validity of approximation and also disagree at some point. Time delay estimation method presented based on correlation method. One of the major advantages this method has is that it achieves better performance when the signals are highly corrupted compared to noise free environment. The proposed method was proven to be effective when compared with theory as presented in the literature and the result of the simulation. Most of conventional methods for estimating the time-delay of the signal to be detected are proposed under the assumption of stationary random noise. On the

contrary to such situation, an approach to the estimation problem of time-delay has been proposed in this thesis under the situation of non-stationary random noise. The usefulness of the approach has been verified by MATLAB simulation results.

5.3 Recommendation

The research work was carried out using linear frequency modulated signal and Wigner-Ville distribution among other form of time-frequency distribution. It is recommended that other form of signals should be used and other forms of signal representation to enable comparison. This research work is recommended for used in the following areas:

(i) Geophysics

(ii) Communication

(iii) Radar etc.

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Appendix A: Matlab Code of The Simulation

```
%
%   Function to generate a chirp signal.
%   function [y]=chirp0(tduration,fsamp,f0,bandwidth)
function [y]=chirp1(tduration,fsamp,f0,bandwidth)
nduration=fsamp*tduration;
alpha=bandwidth/tduration;
y=zeros(1,nduration);

for k=1:nduration
   phase=2*pi*(f0+(alpha/2)*(k-1)/fsamp)*((k-1)/fsamp);
   y(k)=exp(j*phase);
end
```

```
%  Program to implement the smooth windowed Wigner-Ville distr.
%
%  Date : 9/2/2018
%
%  [Pz,Kz]=swwvd0(fsamp,x,alpha,T_win,Tsm,plot_choice),Pz-time-freq.
%  distribution,fsamp-sampling frequency,x-signal,T_win-lag window
%  -width,plot_choice-0 (waterfall),1 (contour),2 (no plot).
%       alpha=1 : Rectangular window
%       alpha=0.54 : Hamming
%       alpha=0.5 : Hanning

function [Pz,Rz]=swwvd0(fsamp,x,alpha,T_win,Tsm,plot_choice)

%   Initialize. %

[num_t]=length(x);
if num_t<512
   num_tau=num_t;
else
   num_tau=512;
end
fmax=fsamp/2;
Rz=zeros(num_tau,num_t);
Pz=zeros(num_tau,num_t);
y=zeros(1,num_t);
yout=zeros(1,num_t);
disp('Start time');
fix(clock)
```

```
%   Calculate the bilinear product. %
ctr=0;
for k=1:num_tau/2
   y=rt_tau2(num_t,ctr,x);
   if ctr==0
      Rz((ctr+round(num_tau/2)),(1:num_t))=y(1:num_t);
   else
      Rz((ctr+round(num_tau/2)),(1:num_t))=y(1:num_t);
      Rz((round(num_tau/2)-ctr),(1:num_t))=conj(y(1:num_t));
   end
   ctr=ctr+1;
end

%   Window the bilinear product.   %

clear y;
for k=1:num_t
   y(1:num_tau)=Rz((1:num_tau),k);
   yout=lagwn1(fmax,alpha,T_win,y);
   for l=1:num_tau
      Rz(l,k)=yout(l);
   end
end

size(Rz);



%   Time-smooth the bilinear product    %

[Rz]=tsmoth(fsamp,Rz,T_win,Tsm);

disp('Stop Time');
fix(clock)

%   Calculate the windowed Wigner-Ville distribution. %

[Pz]=tfdw3d(Rz,fsamp,plot_choice,0);
```

```
% this programme is to estimate the time-delay CC
% using Random noise with zero mean and one variance
signallength=length(mtlb);
assumesignallength=2*signallength;
signal1=zeros(assumesignallength,1);
for n=1:signallength;
signal1(n)=mtlb(n);
end
% assume a tiem-delay;
delay=243;
% generate a time-delay signal;
signal2=lagmatrix(signal1,delay);
signal2(1:delay)=0;
% generate a noise signal;
noise1=randn(signallength,1);
noise2=randn(signallength,1);
for n=(signallength+1):assumesignallength;
noise1(n)=0;
noise2(n)=0;
end
% generate two noisysignal by adding the signal and noise;
signal1=signal1*10;
signal2=signal2*10;
noisysignal1=signal1+noise1;
noisysignal2=signal2+noise2;
% calculate the signal-to-noise ration, SNR
signalpower=0;
noisepower=0;
for n=1:assumesignallength;
signalpower=signalpower+(abs(signal1(n)))^2;
noisepower=noisepower+(abs(noise1(n)))^2+(abs(noise2(n)))^2;
end
```

```
signalpower=signalpower/signallength;
noisepower=noisepower/(signallength*2);
snr=10*log10(signalpower/noisepower);
% Average Square Difference function method;
% change the lag;
for lag=1:400;
delaynoisysignal1=lagmatrix(noisysignal1,lag);
delaynoisysignal1(1:lag)=0;
error(lag)=0;
% calculate the squre error;
for n=1:assumesignallength;
error(lag)=error(lag)+(delaynoisysignal1(n)-noisysignal2(n))^2;
end
end
% find the lag of the minimum error square;
[minimum estiamtion]=min(error);
snr,estiamtion
```